DOCUMENT RESUME

ED 463 610 EC 308 904

AUTHOR	Blank, Shelby L.; Smith, Denise G.; McCombs, Lee Ann
TITLE	Introduction to Algebra. Teacher's Guide. Parallel Alternative Strategies for Students (PASS).
INSTITUTION	Leon County Schools, Tallahassee, FL. Exceptional Student Education.; Florida State Dept. of Education, Tallahassee. Bureau of Instructional Support and Community Services.
REPORT NO	ESE-9412
PUB DATE	1998-00-00
NOTE	143p.; Reprinted in 1998, originally published in 1997. Part of the Curriculum Improvement Project funded under the Individuals with Disabilities Education Act (IDEA), Part B.
AVAILABLE FROM	Florida State Dept. of Education, Div. of Public Schools and Community Education, Bureau of Instructional Support and Community Services, Turlington Bldg., Room 628, 325 West Gaines St., Tallahassee, FL 32399-0400 ($4.30). Tel: 800-487-0186 (Toll Free); Tel: 850-487-0186; Fax: 850-487-2679; e-mail: cicbisca@mail.doe.state.fl.us; Web site: http://www.leon.k12.fl.us/public/pass.
PUB TYPE	Guides - Classroom - Teacher (052)
EDRS PRICE	MF01/PC06 Plus Postage.
DESCRIPTORS	Academic Accommodations (Disabilities); Academic Standards; *Algebra; Curriculum; *Disabilities; *Equations (Mathematics); Inclusive Schools; Instructional Materials; Integers; Mathematics Instruction; *Motivation Techniques; Secondary Education; *Secondary School Mathematics; State Curriculum Guides; Student Motivation; Teaching Guides; *Teaching Methods; Textbooks
IDENTIFIERS	*Florida

ABSTRACT

This teacher's guide contains a collection of supplemental algebra activities adapted for secondary students who have disabilities and other students with diverse learning needs. It consists of alternative methods, materials, and activities that support strands, standards, and benchmarks found in Florida's Sunshine State Standards for mathematics. The activities in the text are intended to actively engage students in hands-on, conceptually-based learning experiences that are grounded in real life. Ideas and strategies are provided for the teacher to motivate students and maintain, assess, and extend the concepts being taught. An answer key for practice pages that do not require student-generated data is included. The guide is divided into 24 separate activities. Topics for the activities include: (1) real-world problems; (2) ordering integers; (3) equivalent forms; (4) algebraic expressions; (5) rational numbers; and (6) relationships, patterns, and functions using words, symbols, variables, and tables. For each activity, the guide includes goals, step-by-step directions, motivation techniques, suggestions for enrichment, and an assessment to measure student performance. Appendices contain a chart describing standards and benchmarks. (Contains 56 references.) (CR)

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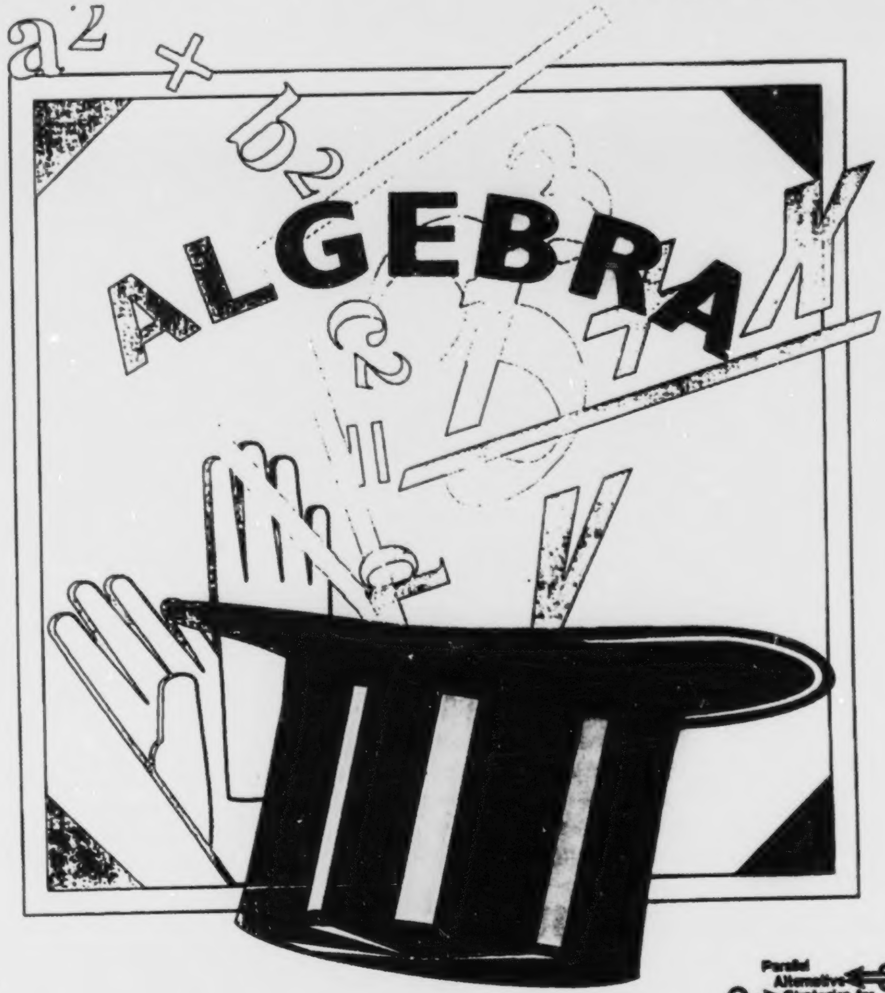
Introduction to Algebra

Teacher's Guide



Parallel
Alternative
Strategies for
Students

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Introduction to Algebra

Teacher's Guide

Bureau of Instructional Support and Community Services
Division of Public Schools and Community Education
Florida Department of Education

Reprinted 1998

This product was developed by Leon County Schools, Exceptional Student Education Department, through the Curriculum Improvement Project, a special project, funded by the State of Florida, Department of Education, Division of Public Schools and Community Education, Bureau of Instructional Support and Community Services, through federal assistance under the Individuals with Disabilities Education Act (IDEA), Part B.

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ALGEBRA

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Teacher's Guide

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IDEA, Part B, Special Project

LEON COUNTY SCHOOLS

Exceptional Student Education

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Acknowledgments

The PASS volume, *Introduction to Algebra: Teacher's Guide*, is a collection of supplemental algebra activities. These activities come from a variety of sources, including a compilation of teacher-adapted activities from *Opening the Gate*, a 1992 publication of the Florida Department of Education, and publishers such as the National Council of Teachers of Mathematics; Scott, Foresman and Company; The University of Delaware; Recycled Paper Products; and others.

The staff of the Curriculum Improvement Project wishes to express appreciation to the curriculum writers and reviewers for their assistance in developing this first edition of the *Parallel Alternative Strategies for Students (PASS)* volume for *Introduction to Algebra: Teacher's Guide*.

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Foreword

Parallel Alternative Strategies for Students (PASS) books are content-centered packages of alternative materials and activities designed to assist secondary teachers to meet the needs of students of various achievement levels in the basic education content courses. Each *PASS* book offers teachers supplementary activities and strategies to assist certain exceptional students and low-achieving students in the attainment of the intended outcomes of a specific course.

The alternative methods and activities found in the *PASS* materials have been adapted to meet the needs of students who have mild disabilities and are mainstreamed in content classes. The *PASS* materials provide basic education teachers with a modified approach for presenting the course content that may be useful with these students and other students who have learning or behavior problems. The *PASS* materials also provide the exceptional education teacher who is teaching subject area courses with curriculum materials designed for these exceptional education students.

Students with learning or behavior problems often require alternative methods of presentation and evaluation of important content. The content in *PASS* differs from the standard textbooks and workbooks in several ways: simplified text, smaller units of study, reduced vocabulary level, increased frequency of drill and practice, shorter reading assignments, clearer and more concise directions, less cluttered format, and the presentation of skills in small, sequential steps.

PASS may be used in a variety of ways. One way to utilize *PASS* may be as material to augment the curriculum for exceptional students and other low-achieving students. For example, some infusion strategies for incorporating this text into the existing program are as follows:

- additional resource to the basic text
- pre-teaching tool (advance organizer)
- post-teaching tool (review)
- alternative homework assignments
- extra credit
- make-up work
- outside assignments
- individual contract
- independent activities for drill and practice
- general resource material for small or large groups
- extension activities for a basic concept
- assessment of student learning.

The initial work on *PASS* materials was done in Florida through Project IMPRESS, an Education of the Handicapped Act (EHA), Part B, project funded to Leon County Schools from 1981–1984. Four sets of modified content materials called *Parallel Alternate Curriculum* (*PAC*) were disseminated as parts two through five of *A Resource Manual for the Development and Evaluation of Special Programs for Exceptional Students, Volume V-F: An Interactive Model Program for Exceptional Secondary Students* (IMPRESS). Project IMPRESS patterned the *PACs* after the curriculum materials developed at the Child Service Demonstration Center at Arizona State University in cooperation with Mesa, Arizona, Public Schools.

A series of nineteen *PASS* volumes was developed by teams of regular and special educators from Florida school districts who volunteered to participate in the EHA, Part B, Special Project, Improvement of Secondary Curriculum for Exceptional Students. This project was funded by the Florida Department of Education, Bureau of Education for Exceptional Students (now the Bureau of Instructional Support and Community Services), to Leon County Schools during the 1984 through 1988 school years. Basic education subject area teachers and exceptional education teachers worked cooperatively to write, pilot, review, and validate the curriculum packages developed for the selected courses.

Beginning in 1989, the Curriculum Improvement Project contracted with Evaluation Systems Design, Inc., to design a revision process for the nineteen *PASS* books. First, a statewide survey was disseminated to teachers and administrators in the 67 school districts to assess the use of and satisfaction with the *PASS* books. Teams of experts in instructional design and teachers in the content area and in exceptional education then carefully reviewed and revised each *PASS* book according to the instructional design principles recommended in the recent research literature.

Neither the content nor the activities are intended to be a comprehensive presentation of any course. These *PASS* materials, designed to supplement textbooks and other instructional materials, should *not* be used alone. Instead, they should serve as a stimulus for the teacher to design alternative strategies for teaching the student performance standards to the mastery level to the diverse population in a high school class.

PASS provides some of the print modifications necessary for students with special needs to have successful classroom experiences. To increase student learning, these materials must be supplemented with additional resources that offer visual and auditory stimuli, including computer software, videotapes, audiotapes, and laser videodiscs.

User's Guide

PASS: Introduction to Algebra is designed as a supplementary resource for teachers who are teaching beginning algebra to secondary students of various achievement levels. It consists of alternative methods, materials, and activities that support strands, standards, and benchmarks found in Florida's *Sunshine State Standards: Mathematics, 1996.* All activities included in *Introduction to Algebra* were adapted from resources developed by *Opening the Gate,* a statewide effort to produce materials for and enhance the pedagogy of those teaching algebra. A matrix that illustrates the correlation between each activity and the *Sunshine State Standards* is included in *Appendix B.*

All activities in *Introduction to Algebra* are intended to actively engage students in hands-on, conceptually-based learning experiences that are grounded in real life. Ideas and strategies are provided for the teacher to motivate students and maintain, assess, and extend the concepts being taught. An *Answer Key* for practice pages that do not require student-generated data is in *Appendix A.* Many of the activities include practice pages that can be copied and used to reinforce the concepts being taught. Many of the activities also include suggestions for integrating technology into instruction. The use of calculators is encouraged.

PASS: Introduction to Algebra is divided into 24 separate activities. Topics for the activities are as follows.

Activities 1-6, 10, 12, 18, 20-24—real-world problems
Activities 7, 13-14—ordering integers
Activity 8—equivalent forms
Activities 9-12—algebraic expressions
Activities 15 and 16—rational numbers;
Activities 17 and 19—relationships, patterns, and functions using words, symbols, variables, and tables.

Each activity is organized in the same way and includes the information teachers need to implement it in their classrooms. The following information is provided for each activity.

Blueprint. Each activity starts with a list of suggested goals that were extracted from the benchmarks in Florida's *Sunshine State Standards: Mathematics, 1996.* The identifying code that corresponds to the code used in that document is in parenthesis at the end of each goal. The wording of each goal is fairly global and teachers are encouraged to use them as a starting point for writing specific objectives that are appropriate for specific classes or individual students.

Key Words. Key vocabulary words are listed for each activity. An *Index to Key Words* is included as *Appendix C*.

Attraction. Each activity includes a strategy for engaging students in the activity. These strategies are called *Attractions* and are a way of capturing students' interests at the beginning of an activity. Attractions vary from activity to activity and include hands-on activities, problems to solve, thought-provoking questions, simulations, and other techniques.

Tools. Materials and other resources needed to implement an activity are listed under *Tools*.

Steps. *Steps* is a step-by-step guide to follow when implementing each activity. It includes specific information on how to do the activity, the order of instructional events, the questions to ask, and other helpful information.

Assessment. Specific assessment strategies and ideas for optional assessment activities are included in the *Assessment* section.

Extension. All of the activities can be modified to fit specific situations or stretched to include more complex ideas or processes. *Extension* includes ideas and options for expanding or changing the scope of the learning activity. Some of the more advanced benchmarks can be addressed by using the suggestions in *Extension*.

Source. This section tells where the activity came from or who originally submitted it to the developers of *Opening the Gate*.

The learning activities are designed to complement current classroom practices. Even though the activities are based on suggested goals and specific concepts, the underlying techniques are often generic enough to be used for teaching other goals and concepts or to be modified to fit the style of individual classrooms. These teacher-developed strategies, coupled with other carefully selected resources, should provide a good foundation in algebraic concepts for students in an exceptional education program and other students with learning problems.

Magic Valley

BLUEPRINT

The student will

- add, subtract, and multiply whole numbers to solve real-world problems, using appropriate methods of computing (MA.A.3.3.3).

KEYWORDS

spreadsheet

ATTRACTION

Students from Valley High spent a day at Magic Valley Amusement Park. Have students compute how much more or less it would cost to take your class on the same trip.

TOOLS

Calculator and/or spreadsheet. Copies of *Practice 1A*, *1B*, and *1C*.

STEPS

- Divide students into cooperative groups. Introduce the Attraction and provide each group with *Practice 1A*, *Magic Valley Information Sheet*: Allow the groups time to read the information.
- Have students complete *Practice 1B* and *1C* to calculate the cost of the Valley High School field trip to Magic Valley. If available, a computer spreadsheet might also be an effective tool.
- When students have completed their study of the Valley High field trip, they can compute the cost of their own trip to Magic Valley. Student groups survey the others in the class to determine the food and drinks selected. Plans should include the number of chaperones and their expenses. Students then complete the same analysis for their class as for the Valley High group.

Magic Valley (continued)

ASSESSMENT

Students write an evaluation of their group's process in determining the cost of their class field trip to Magic Valley.

EXTENSION

This activity could be expanded to include transportation possibilities and cost. The study could also explore fund-raising activities and investigate the percent of profit.

Demonstrate how the calculation of the amount spent for each item (and total amount spent) can be represented by an algebraic equation.

SOURCE

Gladys Thompson, Duval County
Roberta Dilocker, Citrus County

Magic Valley Information Sheet

The honored students on our team enjoyed their trip to Magic Valley Amusement Park, even though they had to leave Moreno Valley at 6:45 a.m. The thirty students, along with Mr. and Mrs. Shultz, Mr. Kobelski, and Mr. Miller, each paid $12.95 to enter the park. Admission to the park included free admission to all rides and shows.

Everyone began the day by riding the Screaming Cyclone ride backwards. Thirteen of the students then rode the Ghost Buster Express, while Mrs. Shultz and four students relaxed and ate bacon crescent rolls for $1.49 each. At the same time, Mr. Kobelski took five students with him to see the F Team Stunt Show; Mr. Shultz and three students rode the Plummeting Parachutes, and Mr. Miller joined the rest of the students on Rambo's Revenge.

At lunch, twelve of the students and Mr. Schultz ate the 79¢ hotdogs, fifteen students and Mr. Miller ate the 89¢ tacos, and the rest of the people bought Super Deluxe Fajitas for $2.29 in the Mexican Village. Everyone drank Magic Colas for 98¢ each.

By the time we arrived back in Moreno Valley at 8:15 p.m., we were all exhausted. Fortunately, we still had enough energy in math class the next day to answer the questions our teacher provided to the class.

Magic Valley Calculation Sheet

Directions: *Use the information on* Practice 1A *to fill in each blank below.*

1. Number of people on trip to Magic Valley ______
2. Price to enter Magic Valley per person $ ______
3. ***Total amount spent to enter Magic Valley*** $ ______
4. Number of people purchasing bacon crescent rolls ______
5. Price for each bacon crescent roll $ ______
6. **Total amount spent on bacon crescent rolls** $ ______
7. Number of people purchasing hotdogs ______
8. Price for each hotdog $ ______
9. **Total amount spent on hotdogs** $ ______
10. Number of people purchasing tacos ______
11. Price for each taco $ ______
12. **Total amount spent on tacos** $ ______
13. Number of people purchasing Super Deluxe Fajitas ______
14. Price for each fajita $ ______
15. **Total amount spent on fajitas** $ ______
16. Number of people purchasing Magic Colas ______
17. Price for each cola $ ______
18. **Total amount spent on colas** $ ______
19. ***Total amount spent on all food and drink*** $ ______
20. ***TOTAL amount spent by class on Magic Valley trip*** $ ______

Magic Valley Worksheet

Directions: *Answer the following questions using the completed calculation sheet.*

1. How much did it cost for all the Moreno Valley group to enter Magic Valley? ______________________________
2. How many people rode the Plummeting Parachutes? ______________
3. How many people rode on Rambo's Revenge? ________________
4. How much money was spent on all drinks? __________________
5. How much money was spent on all food (not including drinks)? ________
6. How much time elapsed from the time the bus left Moreno Valley until the time it arrived back in Moreno Valley? ______________________

   ____
7. How much more money was spent on tacos than on hotdogs? __________
8. Make up five questions of your own based on the information given. _____

   ____

   ____

   ____

   ____

   ____

   ____

   ____

   ____

   ____

Activity 2

Earn Your Way through School

BLUEPRINT

The student will

- add and multiply whole numbers to solve real-world problems, using appropriate methods of computing (MA.A.3.3.3).
- interpret data that has been collected in tables (MA.E.1.4.1).
- represent and solve real-world problems graphically, using algebraic equations (MA.D.2.3.1).

KEYWORDS

invoice

ATTRACTION

Ask: "How much money can a student earn for good grades and excellent attendance?" (See *Practice 2A*.)

TOOLS

Student report cards (real or fictitious). Calculators. Invoice form. Student "check" form. Copies of *Practice 2A* and *2B*.

STEPS

- Students may work individually or in small groups. Give students copies of 10 final report cards (these may be real or made up). Introduce the Attraction and discuss the stipulations of the Lottery winner provided in the information on *Practice 2A*.
- Discuss a sample report card and completed invoice to prepare students for their individual projects.
- Distribute copies of *Practice 2A* and *2B* to each student.
- Students complete their own invoices based on their individual report cards and complete the blank check accordingly.

Earn Your Way through School (continued)

ASSESSMENT

Pairs of students check each others work and resolve errors.

EXTENSION

Students might develop a news release or a letter to the school's superintendent announcing the Lottery scholarships.

Calculate grade point averages and construct a graph of GPAs vs. Total Days Absent.

Discuss how these calculations might be converted into an algebraic equation. Write a computer program to calculate the amount of the award.

S E
OURC

Denise Wilkes, Putnam County

Earn Your Way through School — Practice 2A

Information Sheet

Superior School District in Fantasy, Florida, received $87 million from a lucky Lottery winner who was not so lucky. (He expired from the excitement.) He did, however, have time to write a note bequeathing the great fortune to the students of Fantasy who meet all of these requirements:

- Earned A's in at least three courses
- Have no F's
- Have not missed more than five days

$
$
The Florida Lottery
$
$

The money is divided among those students whose final report cards meet the above requirements according to these guidelines for earnings.

- A's earn $100
- B's earn $50
- no D's earns an extra $100
- no C's earns an extra $50
- no absences earns an extra $200

Earn Your Way through School

Directions: *If you can answer yes to the three questions below, complete the Invoice based on your grades and attendance, and write yourself a check for the correct amount.*

A's in at least three courses? __________

No F's? __________

Absent five days or fewer? __________

EARN YOUR WAY
INVOICE
Superior School District
Fantasy, Florida

Grades/Absences	Yes	No	Number	Calculation	Amount	
A's?				MULTIPLY number X $100		
B's?				MULTIPLY number X $50		
C's?				If *no*, ADD $100		
D's?				If *no*, ADD $50		
Absences?				If *no*, ADD $200		
				TOTAL		

350

Superior School District
Fantasy, Florida

__________ 19 ____

16-66
1220

PAY TO THE ORDER OF __________________ $ ______

__________________ DOLLARS

Citizens State Bank
[illegible]

MEMO __________

James C. Brown
Signature Superintendent

1220 0867 314305519

How Many Bears?

BLUEPRINT

The student will

- relate the concepts of measurement to similarity and proportionality in real-world situations (MA.B.1.4.3).
- collect and interpret data by drawing conclusions based on statistics and tables (MA.E.3.3.1).

KEYWORDS

mean, ecologist, proportionality, zoologist

ATTRACTION

Ecologists and zoologists use the capture/recapture method to estimate the number of animals in the wild. Through a process of tagging and sampling, student scientists will estimate the population of bears in a forest.

TOOLS

Teddy Graham cookies, Teddy Bear pasta, or Goldfish crackers. Plastic bags and cups. Markers. Calculators. Copies of *Practice 3*.

STEPS

- Divide students into groups, and give each group a box or container of bear cookies or pasta. A paper cup can serve as a standardized scoop. For accurate results, students should use a full scoop each time a sample is captured. Students may use the accompanying worksheet to record their data and results.
- Students should first capture a scoop or cupful of bears, count them, tag them using a marker or other means, and put them back in the container and mix well.

How Many Bears? (continued)

- In order to determine the population, students will capture 10 additional samples using the standardized scoop. For each trial sample, they will count the total number of animals and total number of tagged animals, and record these numbers in the data table in *Practice 3*. Between trials, they should return all bears to the container and mix well.

- Students complete the data table, and then calculate the mean of the tagged animals and the mean of the total animals in the sample.

- In order to calculate the total number of animals, students create, discuss, and solve the following proportion.

$$\frac{\textbf{mean number of tagged animals per sample}}{\textbf{mean number of animals per sample}} = \frac{\textbf{original number of animals tagged}}{\textbf{number of animals in population}}$$

- Have students complete *Practice 3.*

Discussion Questions:

1. Were the number of animals in each of your samples about the same?
2. Were the number of tagged animals in each of your samples about the same?
3. What are some animal groups for which this population counting method, capture/recapture, would be appropriate?
4. What are some animal groups for which this population counting method would be inappropriate?
5. Discuss reasons for any inaccuracies or errors using this method for the classroom bear sample? for a sample in the wilds?

How Many Bears? (continued)

ASSESSMENT

Group assessment of process is included on the worksheet.

EXTENSION

Repeat the activity with groups using different sample sizes.

Have groups calculate their percentage of accuracy and then graph the percentage of accuracy as compared with the number in the sample for all groups in the class. (This could be used as an opportunity to teach the use of a spreadsheet program.)

Discuss the sample size used in other data collections such as political or consumer surveys.

Have students investigate and report on other sampling methods.

S E OURC

Merita Miller, Bay County

Population Sample

Directions: *Read each item below. Follow the directions and write the correct answers on the lines provided.*

1. Identify your population: ______________________________

2. Capture a sample from your population. Tag or mark all of the animals which you captured in your first sample. How many did you tag? ________

   Return the tagged animals to the population and mix well.

3. Take 10 samples from the population. Return each sample and mix before drawing the next sample. For each sample, record the total number in the sample and the total number of tagged animals in the sample. Record all data in the following table.

Sample #	Total # Tagged Animals in Sample	Total # Animals in Sample
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
MEAN		

Population Sample (continued)

4. Create the following proportion and solve for the number of animals in the population.

$$\frac{\textbf{mean number of tagged animals per sample}}{\textbf{mean number of animals per sample}} = \frac{\textbf{original number of animals tagged}}{\textbf{number of animals in population}}$$

$$\frac{\boxed{\qquad\qquad}}{\boxed{\qquad\qquad}} = \frac{\boxed{\qquad\qquad}}{\boxed{\qquad\qquad}}$$

5. Count the total number of animals in your population.

   Total number: ______________________________

6. How close were you to estimating the total number in the population using the capture/recapture method?

   ____

7. What is the difference between your estimated population and the actual population?

   ____

8. Divide this number by the actual population and express as a percentage.

   ____

9. Suppose your group is going to repeat the procedure for estimating a population. Describe the methods your group would use to improve your tagging and sampling process.

   ____

   ____

   ____

The Shadow Knows

The student will

- relate the concepts of measurement to similarity and proportionality in real-world situations (MA.B.1.4.3).

ratio

Ask: "Can you determine the height of a telephone pole or tree without climbing to the top or cutting it down?"

Long metric measuring tape. Several small measuring tapes. Sunshine. Tree or telephone pole.

- Divide students into groups of 4. The more varied the heights of the students, the better.
- Introduce the Attraction and challenge students to figure out the answer. (As a clue, you may tell them that it must be sunny.)
- Review the properties of similar triangles.
- Move the class outdoors. Have at least two students from each group measure their height and the length of their shadow. Record all results.
- Measure the length of the shadows of the other two students in the group. Use the measurements in a ratio to predict the height of these students.

$$\frac{\textbf{Student \#1 height}}{\textbf{Student \#1 shadow}} = \frac{\textbf{Student \#3 height}}{\textbf{Student \#3 shadow}}$$

The Shadow Knows (continued)

- Compare predictions to the student's actual height.
- Discuss the ratio of height to shadow found by the groups.
- Next, have students pick a large object nearby (tree, telephone pole, etc.) and measure its shadow length. Use the ratio to determine the height of the object.

Have students write story problems related to the activity. Use some of these problems in a written test.

Compare predicted heights with actual heights of students for different groups.

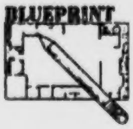
Discuss with students the various circumstances which might affect the outcome of the measurements (e.g., time of day, time of year).

What applications might this activity have in the real world? What occupations might need this skill?

Chris Willis, Hamilton County

Activity 5

Nutty Putty



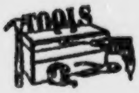
The student will

- relate the concepts of measurement to proportionality in real-world situations (MA.B.1.4.3).
- solve real-world problems involving capacity (M.A.B.1.2.2).
- understand that numbers can be represented in a variety of equivalent forms, including fractions and percents (MA.A.1.4.4).

percent, proportion, ratio

ATTRACTION

Using liquid starch and white glue, students attempt to create the perfect "nutty putty"—a pliable, moldable mixture. Work with students to establish a criterion for perfect nutty putty.



Liquid starch. White glue. Measuring cups or spoons. Food coloring (optional). Clean-up supplies. Copies of *Practice 5*. Nutty Putty, so students can see a finished product.

Give students the following directions.

- You will find some liquid starch and some white glue on your table. By mixing these ingredients together, you can make nutty putty.

- Your task is to determine how much of each ingredient you should use to make perfect nutty putty. Begin with a 50:50 mixture (equal parts of starch and glue) and increase or decrease to reach proper consistency. Knead the mixture following each addition.

Nutty Putty (continued)

- Measure each ingredient carefully and keep a record of the amounts you use so you are able to accurately report to the class what percent of your nutty putty is liquid starch and what percent is white glue.
- Record the total amounts of each ingredient in the chart below. Complete the chart (see *Practice 5*, p. 21) to show the fractional part of the mixture and the percentage of the mixture.

Ask students the following questions:

1. What mathematical principles were learned from this activity?
2. How could you have improved the group process?
3. How well did you work together as a group? Give an example of how your group worked together.

Discuss the difference between the ratio of one ingredient to the other and the ratio of one ingredient to the whole mixture.

Discuss various problem-solving strategies used in the recipe problems. Possibilities include tables, algebraic equations, and the standard equality of ratios (i.e., a:b::c:d).

Adapted from *Opening the Gate*.

Recipe Ratios

Directions: *Using the examples given in the first and second portions of the chart below, complete the chart to show the fractional part of the mixture and the percentage of the mixture for Nutty Putty. Answer the questions that follow.*

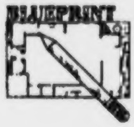
Ingredients	Measurements	Fractional Part	Percentage
Liquid starch	¼ cup	½	50%
White glue	¼ cup	½	50%
Total	½ cup	2/2 = 1	100%
Liquid starch	60 ml	⅓	33⅓%
White glue	120 ml	⅔	66⅔%
Total	180 ml	3/3 = 1	100%
Liquid starch			
White glue			
Total			

1. In your recipe, what is the ratio of liquid starch to white glue? __________
2. Using your recipe, how much liquid starch should be added to 1 cup white glue to make the perfect "nutty putty"?

   ____
3. To ½ cup? ____
4. To 5 cups? ____

Activity 6

The Big Drip



The student will

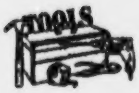
- relate the concepts of measurement to similarity and proportionality in real-world situations (MA.B.1.4.3).
- collect and organize data (MA.E.1.3.1).

variables

ATTRACTION

Present the following situation to the class.

> When you returned from your 14-day vacation, you discovered that you left your bathtub faucet dripping. How many gallons of water were wasted in the 14 days?



Eye dropper. Small paper cups. Water. Measuring cup. Class data poster. Pint, quart, and gallon containers. Copies of *Practice 6A* and *6B*.

- Have students collect data at home by placing a cup under a dripping faucet for 15 minutes and measuring the amount of water collected. If they do not have a cup with ounce markings, they should time how long it takes to collect ¼ cup of water...OR...collect data in class with droppers and paper cups. Ask the students to count the number of drops in one minute and to measure the amount collected.
- Analyze the collected data. Discuss reasons for variability in results. Decide on a reasonable "water drip rate" for the class investigation. Ask students to record an initial estimate of the amount of water wasted in two weeks.

The Big Drip (continued)

- Present some sample problems.

1. If 1 cup of water drips in 1 hour, how much drips in 2 hours? in ½ hour? How long would it take to collect 1 quart (4 cups)?
2. Suppose we find that 5 ounces of water drip in 15 minutes. How many ounces would collect in 1 hour at that rate? How long would it take for 1 cup of water to collect at that rate? (One cup = 8 fluid ounces.) How much water will drip during one day's time?

- Have the class describe their strategies for answering these questions. Some may convert to a rate per unit such as 1 ounce in 3 minutes or 20 ounces in 1 hour. Some may set up a table. Some may use a proportion equation in the $a/b = c/d$ form.
- Refer students to measurement conversion charts. Have the class work in small groups to complete *Practice 6A* and *6B*. Discuss results and compare with original predictions.

Record grades from *Practice 6A* and *6B*.

Use the data collected to solve the following problem.

> How much water would be wasted in a year if each of an estimated 90 million households in the United States had at least one faucet that leaked at this rate? Use a recent utility bill from home to find the cost of the water that leaked during the vacation.

Margaret Longazel, adapted from *Exploring Mathematics:* Grade 6, Copyright © 1991, Scott, Foresman and Company.

The Faucet's Vacation

Directions: *Based on your data collected at home, complete each item below. Write your answer on the lines provided.*

1. How much water do you estimate will drip during the 14-day vacation?

   ____

2. What was the number of drops in 1 minute? ____________________
3. What was the volume of water dripped in 1 minute? ______________
4. How much water leaked in 15 minutes? ____________________
5. How much water leaked in 1 hour? ____________________
6. How much water leaked in 1 day (24 hours)? ____________________
7. How much water leaked in 14 days? ____________________
8. How much water would leak in 1 year? ____________________
9. What factors could account for differences in results among groups?

   ____

   ____

10. The most important thing I have learned from this activity is __________

    ____

    ____

Class Data		
Group	Number of Drops of Water per Minute	Volume of Water per Minute

Step Forward and Take a Bow

BLUEPRINT

The student will

- associate verbal names or written word names with integers (MA.A.1.4.1).
- understand the relative size of integers (MA.A.1.4.2).
- add, subtract, multiply, and divide real numbers including square roots and exponents (MA.A.3.4.3).

KEYWORDS

integer, number line, product, quotient

ATTRACTION

As students enter the classroom, hand each one a card with an integer on it. Have students form a line in the front of the classroom, arranging themselves in order from smallest to largest.

TOOLS

Index cards numbered with consecutive integers from negative to positive (one for each student).

STEPS

After the students have arranged themselves in the correct order, they are asked to translate verbal, mathematical expressions into numbers. Students whose number satisfies the given situation step forward. Some examples of verbal expressions which could be used are provided below. These situations may be made more or less complex depending on individual levels.

Option: Flash written expression on an overhead to accommodate visual learners.

- all numbers greater than 6
- odd numbers
- consecutive even integers starting with –6
- 5 increased by 4

Step Forward and Take a Bow (continued)

- 10 decreased by 3
- 3 decreased by 10
- the product of 2 and 4
- the quotient of 4 and 2
- 6 less than 9
- the opposite of 7
- a number less than 5
- 0 decreased by 8
- 0 increased by 8
- 100% of 1
- 50% of 6
- the sum of −3 and −1
- 2 more than the product of 3 and 4
- the product of 4 and −1, increased by 3
- twice as much as 8
- twice as much as −2
- 15 minus 9
- the quantity negative 3 squared
- 8 fewer than 5
- the difference of 10 and 3

ASSESSMENT

Assess students' understanding by monitoring the accuracy and speed of responses to the situations provided.

Discuss troublesome areas and have each student write an explanation of those terms he or she had trouble with.

Step Forward and Take a Bow (continued)

EXTENSION

Have each student create 10 new example situations for this activity. Encourage students to use a different format, operation, or concept for each example.

Linda Ferreira, Pinellas County
Dorothy Marshall, Lee County

Adapted from Mathematics Curriculum Teaching Program.

Activity 8

High Rollers

The student will

- understand that numbers can be represented in a variety of equivalent forms, including exponents (MA.A.1.4.4).

exponent, power

Ask the following number riddle to introduce students to the concept of powers.

"When are two three's not six?"

Pairs of number cubes (or dice) with both positive and negative numbers. Calculators.

- Ask a student to roll a pair of number cubes. Demonstrate how to write two repeated multiplication problems from the numbers that appear.

  For example: a roll of 3 and 5 could be written as "three, five times or 3 x 3 x 3 x 3 x 3" and "five, three times or 5 x 5 x 5"

- Ask students to use their calculators to calculate the values of the multiplication problems.

  $3 \times 3 \times 3 \times 3 \times 3 = 243$

  $5 \times 5 \times 5 = 125$

High Rollers (continued)

- Explain that these expressions may be written as powers.

  $3 \times 3 \times 3 \times 3 \times 3 = 3^5$

  $5 \times 5 \times 5 = 5^3$

- Have students evaluate expressions a second time by using the y^x key on their calculators.

  $3^5 = 243$

  $5^3 = 125$

- Students may then work in small groups to complete ten trial rolls and record data in a table format as shown.

Roll	Expression
2,1	$2^1 = 2$
	$1^2 = 1$
3,4	$3^4 = 81$
	$4^3 = 64$

- Present the Attraction question to students again:

  "When are two three's not six?"

High Rollers (continued)

ASSESSMENT

Pairs of students may check one another's work and refer their questions to a third student or the teacher.

At the end of the activity discuss which results were most unusual or unexpected.

EXTENSION

Use the same number cubes to find the following:

- two different expressions that have the same value
- the greatest possible product
- the number of expressions that can be written which have a value of 1.

Let the students explore the changes that occur if:

- the first value in a roll is a negative
- the second value in a roll is a negative
- both values in a roll are negative.

Note: This extension is a possible introduction to the meaning of negative exponents.

SOURCE

Bennie Smith, Pinellas County

Algebra War

BLUEPRINT

The student will
- add, subtract, multiply, and divide real numbers using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator (MA.A.3.4.3).
- solve problems involving the algebraic order of operations (MA.A.3.3.2).

KEYWORDS

order of operation

ATTRACTION

Start the class by declaring, "Today we're going to WAR!" Ask students to share what they know about the card game *War.* Explain that this game is similar, but that a "special" deck of cards is used that requires the students to figure out the answer.

TOOLS

Prepare several decks of 48 or 54 cards using blank index cards. Write a mathematical expression on each card. A suggested set of cards (*Practice 9*) follows on pages 37-45. Other cards can be created to suit the level of individual students or to reinforce specific skills. About 10 pairs of cards should have equal values.
Option: calculators.

STEPS

- Divide students into small groups or pairs to play *Algebra War*.
- Dealer distributes an equal number of cards face down to all players. (Leftover cards go to the winner at end of the game.)
- Players turn over their top card, calculate the value of the expression, and announce the answer. The player with the highest value wins that hand and takes all the cards played.

Algebra War (continued)

Note: Play could be changed so the player with the lowest value wins the hand.

- Challenge: Any player may challenge another's answer. The group determines the correct answer. If correct, challenger earns a card from the deck of challenged student.
- If a tie occurs, war is declared. Tied players turn over the next card in their decks, find the values, and determine the winner who takes all the cards played.
- The winner of the game is the person who gets all the cards, or, if a time limit is set, the player who has the most cards when time is called.

At the end of class, identify trouble spots and make sure these are understood.

Give written test with similar items.

Ask students to create five different but equivalent cards.

Students may create their own card decks.

Activity may be continued in learning center. Deck could be devoted to equations, exponents, or other algebra topics.

Jo Ann Morris, Charlotte County

Algebra War Cards

-2^4

$(-2)^4$

$4x^2-5x$
if $x = 3$

$-9-(-3+5)$

$20-(-3)$

$19-27 \div 9$

Algebra War Cards

$\dfrac{45}{5-14}$	$\dfrac{8\,(-3)}{-6}$
$18 + 7 \bullet 4 - (-5)$	$5^2 + 13 + 10^2$
π	$+\sqrt{289} + 3$

Algebra War Cards

$48 \div 16$

$$\frac{(-5)^2 + 2}{-3}$$

largest prime less than 100

largest multiple of 7 less than 50

$2^3(-2)^3$

$+\sqrt{625}$

Algebra War Cards

$3 \bullet 14$	$\frac{-100}{-4}$
the reciprocal of -1/45	the reciprocal of 7/91
$8 + (1)^4$	$-36 \div 2 + 3$

Algebra War Cards

$720 \div 10 \div -9$

$\frac{-80}{-40} - \frac{60}{30}$

the opposite of 20

the side of a square whose area is 49

$3^3 - 7(3)$

$-3(-19)$

-81 ÷ -9(4)

$$\frac{36 + -45 + -9}{-3}$$

⅓ of a right angle

the side of a square whose perimeter is 20

-54 ÷ 9 + 6(20) ÷ 2

4(7 + 5) – 6(8)

Algebra War Cards

the number of degrees in a triangle minus 42

the number of degrees in the other acute angle of a right triangle if one angle is 55 degrees

$$\frac{\text{the number of degrees in a circle}}{3}$$

a perfect square between 80 and 90

the reciprocal of 2/3 x -14

|-4| + 19 – 5

Algebra War Cards

$-8(9) - 3$

The least prime number

$-7(2^4)$

$-3^2 + 7$

The number of distinct whole number factors of 36

The value of x if $3x = -39$

Algebra War Cards

$$-2(5 - 11)$$

$$\frac{2 + 5(6)}{12 + 4}$$

$$10^2 - 3$$

$$-|-15| + 7$$

$$23 + 7(-7)$$

In a circle, the result when the circumference is divided by the diameter

Language Concentration

The student will

- represent problems with algebraic expressions (MA.D.2.3.1).

equivalent expressions

Present students with the question, "How is it possible for 3a to equal 6?"

Concentration game cards made from posterboard. Sample game cards are provided in *Practice 10A* and *10B*. These may be enlarged, laminated, and cut out for use.

This activity is designed to help students avoid misconceptions commonly made when interpreting the language of algebra.

The data table below lists some of the most common misconceptions in using algebraic procedures.

Problem	Equivalent Expression	Misinterpretation
If $a = 2$, then $3a =$	$3 \cdot a$	32
a^3	$a \cdot a \cdot a$	3a
4½	4 + ½	2
$(5x)^2$	$5 \cdot 5 \cdot x \cdot x$	$5 \cdot 5 \cdot x$
$2a + b$	$2 \cdot a + b$	$2 \cdot a \cdot b$
If $x = 5$, then $x^2 =$	$5 \cdot 5$	$5 \cdot 2$

Language Concentration (continued)

- Discuss the equivalent expressions and misinterpretations listed in the data table. Ask for others the students may have experienced, and how they resolved their difficulties.
- Some of the problems, equivalent expressions, and misinterpretations from the data table have been included in a concentration game format. (See *Practice 10A* and *10B* on pp. 49-50.)
- Divide students into groups of four, with two partners challenging the other two. Players shuffle the cards and play them face down in a 5 by 5 array. The teams alternate, turning up two cards at a time in order to locate equivalent expressions. If a match is made, the team removes those cards and takes another turn. Play continues until all possible matches have been made. Each card is worth one point. The remaining card should not match any other card.

Each student identifies his or her troublesome misconceptions.

Follow up with written test on similar algebraic expressions.

This activity could be extended to include negative integers. The difficulty of the game may be adjusted by increasing the size of the array.

Merita Miller, Bay County
Bernedette Meglino, Marion County

Sample Concentration Game Cards

3a	3 • a	30 + 2	32	If a = 2, then 3a =
3 • 2	4½	4 + ½	4(½)	2
If x = 4 and y = 5, then xy =	4 • 5	45	40 + 5	3 + 2
7b	7 • b	If b = 3, then 7b =	7(3)	If ab is 3 • 5, then ba is
5 • 3	12½	10 + 2 + ½	53	50 + 3

Sample Concentration Game Cards

If $x = 5$, then $x^2 =$	25	10	$5 \bullet 2$	$5x^2$
$5 \bullet x \bullet x$	$(5x)^2$	$5 \bullet 5 \bullet x \bullet x$	x^3	$x \bullet x \bullet x$
$3x$	$3 \bullet x$	$2a + b$	$2 \bullet a + b$	$2ab$
$2 \bullet a \bullet b$	If $a = 3$ and $b = 4$, then $2ab =$	$2 \bullet 3 \bullet 4$	234	If $x = 3$ and $y = 4$, then $xy =$
$3 \bullet 4$	34	$30 + 4$	3^3	$3 \bullet 3 \bullet 3$

Activity 11

Magic Squares

BLUEPRINT

The student will

- describe, analyze, and generalize relationships and patterns using words, symbols, variables, and tables (MA.D.1.4.1).
- add and subtract whole numbers (MA.A.3.3.3).

KEYWORDS

matrix

ATTRACTION

Challenge students to create a 3 X 3 magic square. Explain that a magic square is one in which the sum of each row, column, and diagonal set of numbers is the same. Students may use any numbers they choose, but numbers may not all be the same.

TOOLS

Design for a 3 X 3 matrix. Calculator. Transparency of secret code. Copies of *Practice 11*.

STEPS

- Begin class with the Attraction. Provide students with empty 3 X 3 matrix and allow approximately 5 minutes for students to attempt to create a magic square. During this time, circulate and check for understanding. Some students may experience some frustration and need to be motivated and encouraged.
- When the time is up, ask for and discuss any successful magic squares. To insure that the class understands the concept, complete one square as a group. Present the class with the following magic square, and ask what the sum of all the rows, columns, and diagonals should be. Complete this square as a class, demonstrating how to work backwards to find the remaining numbers.

Magic Squares (continued)

2	8	8
	6	

- Next, explain that there is a secret code that can be used to create magic squares. Provide students with a copy of the following array. (See *Practice 11A*.)

$x - z$	$x + z - y$	$x + y$
$x + y + z$	x	$x - y - z$
$x - y$	$x + y - z$	$x + z$

- Explain that the letters *x*, *y*, and *z* represent three numbers. Ask students to pick a value for *y* and a value for *z*. Then have students pick a value for *x* that is greater than *y* and *z*. Students can then create their own magic squares, substituting their chosen numbers in place of the appropriate variable, and calculating the value of the expression. Encourage students to make a chart of the *x*, *y*, and *z* values used and the common sum for each magic square. What generalities can they make? Discuss why the pattern exists.

Magic Squares (continued)

- Predict: If x is 20, what will be the common sum of rows, columns, and diagonals?

Hold each student responsible (in a math journal or portfolio, or as an assignment) for the construction of a 3 x 3 magic square plus a written explanation of why the algebraic pattern works. Include a magic square with missing quantities on a unit quiz.

Ask students to create a magic square in a 4 X 4 matrix. Challenge students to find a secret code to create 4 X 4 magic squares. Ask them to explore whether x must be greater than y and z.

Have students investigate and report on other types of magic squares and patterns for creating them.

Anne Patterson, Volusia County

Adapted from Volusia County School's *Pre-Algebra Preparation Packet.*

Magic Square Equation

$x - z$	$x + z - y$	$x + y$
$x + y + z$	x	$x - y - z$
$x - y$	$x + y - z$	$x + z$

Activity 12

A Magical Mathematical Birthday Card

BLUEPRINT

The student will

- use algebraic problem-solving strategies to solve real-world problems (MA.D.2.3.2).
- describe, analyze, and generalize relationships, patterns, and functions using variables (MA.D.1.4.1).

KEYWORDS

variable

ATTRACTION

Students celebrate a birthday with a bit of mathematical magic. By following the step-by-step instructions on the birthday card, each student ends with a solution that is his or her birth date.

TOOLS

Calculator. Birthday card. Copies of *Practice 12* with instructions.

STEPS

- Inside a birthday card are mathematical instructions to be followed by the person who receives it. Have students follow the instructions step-by-step.

  1. Write the number of the month you were born or enter it in your calculator.
  2. Multiply by 4.
  3. Add 13.
  4. Multiply by 25.
  5. Subtract 200.
  6. Add the day of the month on which you were born.
  7. Multiply by 2.

A Magical Mathematical Birthday Card (continued)

8. Subtract 40.
9. Multiply by 50.
10. Add the last two digits of birth year.
11. Subtract 10,500.

- Discuss your results. Did everyone get his or her date of birth? Why does this work?
- Complete *Practice 12* on p. 58 to derive an algebraic expression for these instructions.
- Show that the instructions given are equivalent to an algebraic expression that will result in the date of birth for anyone.

ASSESSMENT

Check the algebraic expressionby substituting the appropriate numbers for the variables m, d, and y.

EXTENSION

Members of your family, relatives, or friends might be amazed by this birthday trick. Try it on them. After demonstrating it, try to explain it well enough for them to understand it.

In many of the countries of the world, people write dates in this order: day, month, year. That is, instead of writing "October 4, 1993," they write "4 October 1993." This is done in some cases in the United States. How would you change the instructions on the birthday card to fit this way of stating dates?

There are other countries in which people give dates in this order: year, month, day. How would you change the instructions to fit this system?

A Magical Mathematical Birthday Card (continued)

EXTENSION

People in other countries would argue that either of the above ways of writing dates is more logical than that commonly used in the United States. What might their argument be? (Hint: Days, months, and years are time periods of different lengths.)

Make up a completely new set of directions that will result in the same algebraic expression arrived at earlier, so that you can construct your own birthday card.

Dr. Donovan R. Lichtenberg
University of South Florida
Tampa, Florida

Adapted from *Opening the Gate*.

A Magical Mathematical Birthday Card Practice 12

Computation Worksheet

Directions: *In order to develop an algebraic expression that will work for any person, we must use variables rather than specific numbers. It makes sense to use* **m** *to represent the number of the month in which a person was born,* **d** *for the day of the month, and* **y** *for the year number (last two digits). Now fill in the blanks below with algebraic expressions using the instructions and using variables when necessary. The first 3 blanks are filled in for you.*

1. Write the number of the month. m
2. Multiply by 4. $4m$
3. Add 13. $4m + 13$
4. Multiply by 25. ______

   (Apply the distributive property and simplify.) ______
5. Subtract 200 (and simplify). ______
6. Add the day of the month. (Use a variable.) ______
7. Multiply by 2. ______

   (Apply the distributive property and simplify.) ______
8. Subtract 40 and simplify. ______
9. Multiply by 50. ______

   (Apply the distributive property and simplify.) ______
10. Add the year number. (Use a variable.) ______
11. Subtract 10,500 and simplify. ______

To check: Substitute your values for *m*, *d*, and *y*. This answer should be the same as your answer to #11 on the *Computation Worksheet*.

Activity 13

The Human Coordinate Plane

BLUEPRINT

The student will

- identify and plot ordered pairs in four quadrants of a rectangular coordinate system (graph) and apply simple properties of lines (MA.C.3.3.2).
- analyze functions using variables and graphs (MA.D.1.4.1).

KEYWORDS

coordinate plane

ATTRACTION

Create a large, classroom-sized coordinate plane. Students physically plot ordered pairs and become part of the "human coordinate plane."

TOOLS

Wide and narrow masking tape. 25 large index cards. Coordinate grid on overhead projector or chalkboard.

STEPS

- Arrange 25 desks in a square array, or clear desks from the center of room and create a large coordinate plane on the floor using masking tape. Use wide masking tape to represent the axes.
- Label 25 large index cards with the following ordered pairs:

(–2, 2)	(–1,2)	(0,2)	(1, 2)	(2,2)
(–2, 1)	(–1,1)	(0,1)	(1, 1)	(2,1)
(–2, 0)	(–1,0)	(0,0)	(1, 0)	(2,0)
(–2,–1)	(–1,–1)	(0,–1)	(1,–1)	(2,–1)
(–2,–2)	(–1,–2)	(0,–2)	(1,–2)	(2,–2)

The Human Coordinate Plane (continued)

- Project a coordinate plane on an overhead projector or chalkboard. Discuss the origin, x–axis and y–axis. Introduce the concept of ordered pairs (x,y). Beginning with positive numbers, plot sample ordered pairs on the plane as a class. Progress to pairs using negative numbers, and discuss the quadrants in which these pairs are located.

- Pass out the ordered pair cards to students in random order. Explain the orientation of the floor plane, pointing out the origin, x–axis and y–axis. Reference the coordinate plane on the board, if needed. Ask students to place themselves in the desk or on the spot on the floor grid corresponding to the ordered pair shown on their card. (Students may be divided into groups or teams, with points awarded for each correct answer). Demonstrate plotting beginning at the origin and walking off the x and y coordinates.

- Ask all students whose ordered pair card has 0 as the first number to stand on the correct point on the plane. Through discussion, identify 0 as the x coordinate and point out that the students standing form the y–axis. Thus, ordered pairs whose x coordinate is equal to 0 form the y–axis. The equation of this line is $x = 0$. Identify this line as a vertical line.

- Follow the same procedure for students whose ordered pair has 0 as the second number, forming the x–axis. Lead students to the equation $y = 0$. Identify this line as a horizontal line.

- Ask each student with an x coordinate of 1 to stand up on the coordinate plane. Write $x = 1$ on the board. Repeat the procedure for students with an x coordinate of -2. Through discussion, lead students to these conclusions.

  The equations represent lines which are vertical.

  The equations represent lines which are parallel to the y–axis and to one another.

The Human Coordinate Plane (continued)

STEPS

- Follow the same procedure for students with y coordinates of 1 and -1. Through discussion, show that the resulting equations, $y = 1$ and $y = -1$, form lines which are horizontal, or are parallel to the x -axis and to one another.
- Ask students whose ordered pair has a sum of 1 to stand and write $x + y = 1$. These students should remain standing while seated students investigate whether their coordinate pair meets the requirements of the equation $x - y = 1$. Write $x - y = 1$ on the board, and ask students to subtract the second coordinate from the first coordinate. Students who obtain 1 as the result also stand. Through discussion, lead students to see that (1,0) is a point on both lines and represents the point of intersection. Substitute values in the equations on the board to show that (1,0) makes both $x + y = 1$ and $x - y = 1$ true.
- Repeat the above process using $x + y = 1$ and $x + y = 2$. Guide students to discover that if there is no point of intersection, the lines are parallel.

ASSESSMENT

Students are assessed on their ability to position themselves correctly on the plane and their ability to correctly identify the equations of horizontal and vertical lines.

EXTENSION

Ask students whose ordered pair sum is 2 to raise their hands. Now ask students whose ordered pair sum is ***less than 2*** to stand and write $x + y < 2$ on the board. Show the students a graph with a dotted line for $x + y = 2$ and shading for $x + y < 2$. Note that the shading includes all points, not just integral values. Repeat the process for other inequalities.

The Human Coordinate Plane (continued)

EXTENSION

Use an overhead graphing calculator to demonstrate equations used in class and others suggested by students. Challenge students to identify equations that result in intersections and in parallel lines.

Anne Patterson, Volusia County

Activity 14

Picture This

BLUEPRINT

The student will

- understand geometric concepts such as *congruency*, *similarity*, and *transformations* including slides and enlargements (MA.C.2.4.1).
- identify and plot ordered pairs in all four quadrants of a rectangular coordinate system (MA.C.3.3.2).
- describe, analyze, and generalize patterns using graphs (MA.D.1.4.1).

KEYWORDS

quadrant, *x* coordinate, *y* coordinate

ATTRACTION

Explain to students that they are going to complete picture graphs by plotting points. After students are comfortable with plotting in all four quadrants, ask them: "What would happen if you double each coordinate?" and "What would happen if you added one to each coordinate?"

TOOLS

Set of coordinates to make picture graphs (some should be in the first quadrant only). Graph paper. Overhead transparencies with coordinate grid. Various colors of overhead pens (optional).

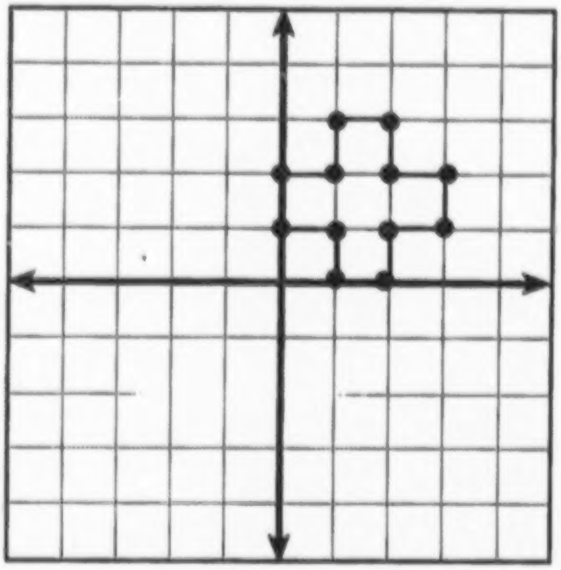
STEPS

- Students may work on this activity in pairs or individually. Give each pair of students the directions for creating a picture graph. Explain that by plotting the given points and connecting them in order, a shape will be created.

Picture This (continued)

For example:

(0,1)
(0,2)
(1,2)
(1,3)
(2,3)
(2,2)
(3,2)
(3,1)
(2,1)
(2,0)
(1,0)
(1,1)
(0,1)



- Give each pair of students the coordinates for the same picture. Then give each group a different rule to apply to the original graph. Explain that by applying this rule, the graphed shape will change. The following are examples of rules which may be used.

Group A	Draw the picture as given.
Group B	Add 1 to the *x* coordinates.
Group C	Add 2 to the *x* coordinates.
Group D	Add 4 to the *x* coordinates.
Group E	Add 1 to the *y* coordinates.
Group F	Add 2 to the *y* coordinates.
Group G	Add 4 to the *y* coordinates.

Picture This (continued)

Group H	Add 1 to both the *x* and *y* coordinates.
Group I	Subtract 2 from the *x* coordinate.
Group J	Subtract 4 from the *y* coordinate.
Group K	Subtract 4 from both coordinates.
Group L	Multiply the *x* coordinates by 2.
Group M	Multiply the *y* coordinates by 2.
Group N	Multiply both coordinates by 2.
Group O	Multiply both coordinates by -2.

- Ask students to draw their graphs on transparencies. Place the original graph on the overhead projector, then ask students to place theirs on top one at a time. What observations can be made? (It may be helpful to begin with simple pictures before moving to more complex ones.)
- Repeat this activity, changing the rules from addition to multiplication. What happens with multiplication? How is this the same or different from addition? What effect would subtracting or dividing have on the graph?
- Later, when students graph equations on a coordinate plane, you might refer again to this general concept, likening it to changing coefficients and constants.

Ask students to explain, in their own words, the generalizations they observed when adding, subtracting, multiplying, or dividing the *x* and/or *y* coordinates.

Picture This (continued)

EXTENSION

Show students two graphs of the same picture, one of which has altered parameters. Ask the students to determine the rule.

Use subtraction and division for the rule operators.

Use negative integers to multiply the x and/or y coordinates.

Use geometric terms such as *congruency*, *similarity*, and *transformations* including slides and enlargements.

Have students create their own, more complex, graph pictures.

Sue Burns, Orange County

Where in the Venn?

BLUEPRINT

The student will
- understand the relative size of rational numbers (MA.A.1.4.2).
- describe a wide variety of relationships through models (MA.D.1.3.1).

KEYWORDS

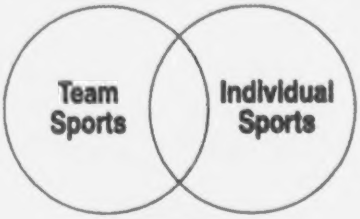
Venn diagram

ATTRACTION

As students enter the class, hand each one an index card with a different sport written on it. Challenge students to place their card in the proper section of the Venn diagram.

TOOLS

Large Venn diagrams. Prepared index cards. Copies of *Practice 15.*



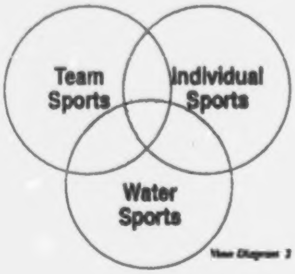
Venn Diagram 1

Suggested Card Topics:

400-meter relay
cross country
football
basketball
volleyball
archery
50-meter freestyle
shot put
bowling
badminton
tennis
hockey
marathon
5-K run
diving
800-meter race
triathlon
jogging
sailing
soccer
race walking
lacrosse
gymnastics
2000-meter medley relay

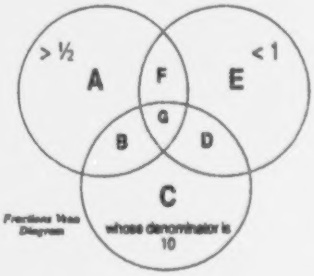
Where in the Venn? (continued)

- After students have placed their cards, discuss the concept of the Venn diagram and what each region represents, allowing students to revise their original placement of the sports cards. Allow students to discuss and defend their answers.
- Repeat this activity, adding a third circle to the Venn diagram. (*See Venn Diagram 2.)* Display the new diagram and ask students to describe what each region represents. Redistribute the index cards, and ask students to place them in the proper region.



Venn Diagram 2

- Introduce the *Fractions Venn Diagram* below in several stages. First present regions A, E, and C as three individual circles, and discuss which fractions would fall in each region. Ask students for any fractions that could be placed in more than one of the circles. Explain that these fractions would fall into the overlapping areas. Have the students identify fractions that belong in each area.



Fractions Venn Diagram

Where in the Venn? (continued)

- Students should then complete *Practice 15*. Check and discuss in class.
- Discuss other topics that could be represented in a Venn diagram. Have students make a two- or three-circle Venn diagram for some mathematical topic of their choice and place one example in each region.

Record scores from completed Practice page.

Have students write a paragraph identifying difficulties and/or successes they encountered with this activity.

Give students decimal fractions to place in the Venn diagram.

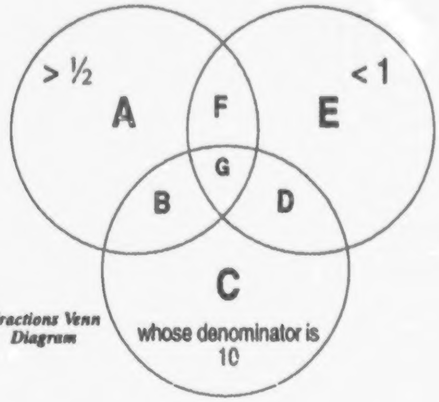
Make a Venn diagram for geometric figures (e.g., quadrilaterals, parallelograms, squares) or for kinds of numbers (e.g., rational, irrational, and real; integers, odd and prime numbers).

Linda Ferreira, Pinellas County
Dorothy Marshall, Lee County

Where in the Venn? Practice 15

Venn Diagram

Directions: *Answer the following questions about the Venn diagram shown below.*



Fractions Venn Diagram

For each fraction, write the letter of the correct region of the Venn diagram.

1. $7/8$ ____________________
2. $5/10$ ____________________
3. $13/10$ ____________________
4. $5/4$ ____________________
5. $7/10$ ____________________
6. $1/4$ ____________________
7. Is there any region into which we cannot place a fraction? ____________
8. Can you think of a fraction that cannot fit into any of the regions? ______
9. Wherever possible, list a different fraction that will fit into each of the following regions.

   A: ____________________

   B: ____________________

   C: ____________________

   D: ____________________

   E: ____________________

   F: ____________________

This Town Ain't Big Enough for All of Us...or Is It?

The student will

- associate verbal names with rational numbers (MA.A.1.4.1).
- understand symbolic representations of rational numbers (MA.A.1.3.3).
- understand the relative size of rational numbers (MA.A.1.4.2).
- understand the basic concepts of limits and infinity (MA.A.2.4.1).
- describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, and tables (MA.D.1.4.1).

denominator, number line, population

Introduce "Fraction Town" to the students. Explain to them that the space between 0 and 1 on the number line is a town, and the fractions between 0 and 1 are the people. Ask how many people live in the town if the denominators are less than or equal to 10.

A large number line from 0 to 1 that will stretch across the front of the classroom (may be drawn on computer paper). A "Fraction Town" sign. Index cards labeled with the following fractions through denominator 10 (one for each student): $1/2$, $1/3$, $2/3$, $1/4$, $2/4$, $3/4$, $1/5$, $2/5$, $3/5$, $4/5$, $1/6$, $2/6$, $3/6$, $4/6$, $5/6$,.... Poster-size blank data table. Calculators. Copies of *Practice 16*.

Note: Students may create cards as the activity develops.

This Town Ain't Big Enough for All of Us...or Is It?
(continued)

- Prepare the classroom in advance by posting the Fraction Town" sign, the large number line, and a blank data table to record results of activity.

- Label enough index cards with fractions for each student to have at least one.

- Introduce the Attraction for "Fraction Town." Ask students to provide guesses as to how many people they think live in the town. (Remember the denominator must be less than or equal to 10.)

- Pass out the fraction cards to students in random order, and explain that each student is moving to "Fraction Town." Begin by asking the student whose fraction card has 2 as the denominator to find "home" on the number line.

- Ask students to record information on the blank class data table as different fraction families move into town (a partially completed data table is provided on page 74 for reference). Guide the class through the first few examples and encourage students to find the rule as they work through the table. (The total population rule is $y = .5x\,(x - 1)$ where x is the denominator.)

- Ask students whose fraction cards have denominator 3 to move into town. Record the information on the table. Students should note that when the fraction families with denominator 4 move in, one will be a member of the 2 family and will live in the same "home" ($\frac{2}{4} = \frac{1}{2}$).

- Continue until all students have moved into Fraction Town. In most cases, classes will not have enough students to move in past the denominator 8. Students identify the pattern in the table and complete the information for denominators 9 and 10.

Tip: To help students visualize, encourage them to fold a strip of paper into thirds, fourths, etc., and label points as they proceed.

This Town Ain't Big Enough for All of Us...or Is It?
(continued)

Observe placement of fractions on the number line and completion of individual data. Ask successful students to explain how they determine placement. Have students write a paragraph explaining what they have learned from the activity and why the town holds an infinite number.

Ask students to find the rule and predict the population when the denominator goes to 12, to 100, to 350, and to any number x. Ask students how many fractions fall between 0 and 1. Discussion should lead students to the conclusion that the town can hold an infinite number of fraction people.

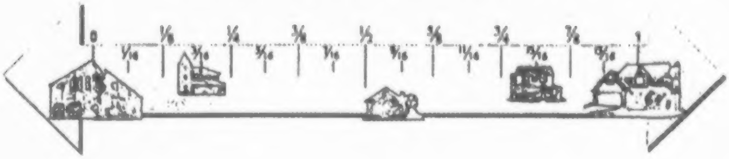
Tony Tangaro, Brevard County

Adapted with permission from "A Population Explosion of Rational Numbers," from *Exploratory Problems in Mathematics* by Frederick W. Stevenson, copyright 1992 by the National Council of Teachers of Mathematics. All rights reserved.

Fraction Town

Directions: *Record the number of people moving into Fraction Town by completing the Data Table below.*

Fraction Town Number Line


0
1/8
1/4
3/8
1/2
5/8
3/4
7/8
1
1/16
3/16
5/16
7/16
9/16
11/16
13/16
15/16

Fraction Town Data Table

Fraction Card Denominator	Number of Fractions	Fraction Town Population
2	1	1
3	2	3
4	3	6
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

Activity 17

What's the Elur?

The student will

- describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, and tables (MA.D.1.4.1).

two-step equation, x = sentence, y = sentence

In this activity students find the rule relating x and y values, then write y = sentences as a "rule," and write x = sentences as an "elur."

Explain to the students that "elur" is "rule" spelled backwards.

Ask students to give you a 1-digit number to record in the x column of an (x,y) table. Tell them you are going to use a "rule" to generate a corresponding number for the y column. Continue to compute a y value for each x given until students are able to guess the rule. Repeat the process using progressively more complex rules. Each time a rule is discovered, the teacher should write it algebraically, explaining the x and y variables.

Overhead transparency for recording numbers in an (x,y) table.

- Have students generate solutions in pairs or cooperative groups.
- Begin using a simple rule such as $y = x = 3$. For each x provided by students, calculate the y value. Record these on the overhead table. When a group indicates they know the rule, check them by providing an x value and asking them to

What's the Elur? (continued)

calculate the correct y value. Explain how to write the rule algebraically.

- When students are comfortable with simple rules using addition, progress to rules using multiplication such as $y = 2x$.
- Explain to students that a rule may use more than one operation, such as addition and multiplication. Such a two-step equation would be: multiply the number by 2 and add 3, or $y = 2x + 3$
- Record student numbers in the x column of an (x,y) table and fill in the y column using $y = 3x + 1$ as your rule.
- Have groups generate a table of x, y values and challenge others to find their rule.
- When students are comfortable generating rules, explain a $y =$ sentence (rule) and its corresponding $x =$ sentence (elur). Demonstrate how one can be derived from the other.

Rule	Elur
$y = x + 3$	$x = y - 3$
$y = 3x$	$x = y/3$
$y = 3x + 1$	$x = \frac{y - 1}{3}$

- Further discussion should lead students to conclude that a two-step equation (rule) can be solved by writing a two-step "elur." Similarly the solution for a one-step rule can be written as a one-step "elur."

What's the Elur? (continued)

Observe groups working on generating tables and finding the rules.

Include class-generated tables in unit quiz.

Have students find examples of real-world situations that are described by two-step equations (e.g., postage rates, long distance telephone rates, cab fares, base salary and commission).

For the following equations, have students complete an (x,y) table for x values 1 to 5:

1.) $y = x - 5$

2.) $3x = y$

3.) $x = \frac{y + 4}{2}$

Generate (x,y) tables and ask students to identify the rule used and write the corresponding algebraic equation.

Lee Bailey and Tony Tangaro, Brevard County

Summer Earnings / Pick a Job

BLUEPRINT

The student will

- add and multiply whole numbers to solve real-world problems, using appropriate methods of computing (MA.A.3.3.3).
- describe, analyze, and generalize relationships and functions using words, symbols, variables, tables, and graphs (MA.D.1.4.1).
- interpret data that has been organized in tables (MA.E.1.4.1).

KEYWORDS

accumulated, bonus, *x*–axis, *y*–axis

ATTRACTION

Explain to the students that they are about to begin a 12-week vacation and they have received two job offers. The first job is at Lee's Lunchorama, which won't be open the first two weeks of vacation. This job guarantees $6 an hour and a 40-hour week.

The second job offer is from Suzie's Sandwiches and Such. This job pays $4.25 an hour and guarantees a 40-hour week. In addition, Suzie offers a signing bonus of $150 if you start work immediately. Which job will they take?

TOOLS

Calculators. Graph paper. Spreadsheet program (optional). Copies of *Practice 18*.

STEPS

- Present students with the Attraction. Before making any calculations, ask students which position they would choose in order to make the largest total salary.

Summer Earnings / Pick a Job (continued)

- Give students the *Accumulated Salary Table, Practice 18.* Discuss methods of completing this table. (Remind students that the first week's pay at Suzie's includes the salary and $150 bonus.)
- Ask students to complete the *Accumulated Salary Table* to discover which job yields the largest payoff.
- Approach the information graphically. Discuss which information should be represented on the x–axis and which on the y–axis. Ask students to label each axis. Discuss the concept of the scale of an axis, giving examples of different scales on a coordinate plane. Choose an appropriate scale for the graph of this data and label the axes accordingly.
- Ask students to plot the data contained in the *Accumulated Salary Table* on graph paper.
- Discuss the graphs representing each of the job opportunities. At what point during the summer would the salaries from the two jobs be equal?
- Discuss factors other than salary to be considered when taking a job.

ASSESSMENT

This activity could be evaluated as a group task with various points awarded for completion of the table, a graph of the information and $y = mx + b$ sentences for each of the jobs. Discuss various approaches to solving this problem.

Summer Earnings / Pick a Job (continued)

EXTENSION

Ask students to create an algebraic equation to represent the salary earnings for each position.

Convert the equations into the $y = mx + b$ form. Discuss the constants in this equation. What does m represent? What does b represent?

Tony Tangaro and Lee Bailey, Brevard County

Adapted from *A Cooperative Investigation Using Math Connections, Algebra I Connectors Activity*. Spring/Summer, 1992.

Accumulated Salary

Directions: *Complete the chart below and answer the questions.*

Accumulated 12-Week Salary		
Week	**Lee**	**Suzie**
1	$ 0	$ 320
2	$ 0	$ 490
3	$ 240	$ 660
4	$ 480	$ 830
5		
6		
7		
8		
9		
10		
11		
12		

1. Which job yields the largest payoff? ____________________

2. Is there a time during the summer when the two accumulated salaries are equal?

____

Activity 19

Probability Cubed

BLUEPRINT

The student will

- determine probability using counting procedures and tables (MA.E.2.4.1).
- determine odds for and odds against a given situation
- compare experimental results with mathematical expectations of probabilities (MA.E.2.3.1).
- understand the basic properties of, and relationships pertaining to, regular geometric shapes in three dimensions (MA.C.1.3.1).
- describe, analyze, and generalize relationships and patterns using words, symbols, variables, and tables (MA.D.1.4.1).

KEYWORDS

cube, face, probability

ATTRACTION

Present the following activity to the class.

> Paint, in your imagination or for real, a 4-inch cube (4-cube) using your favorite color. Cut it into 64 one-inch cubes (1-cube) and select one of them at random. Predict the probability that if you toss it like a die, the top face of the 1-cube will be painted your favorite color.

TOOLS

At least 8 one-inch cubes per group. Calculators. Colored stick-on circles (to simulate painting). Quantity of cubes such as multi-link or unifix cubes. Copies of *Practice 19*.

STEPS

- Introduce the problem and have each student guess the probability of choosing a painted cube.

Probability Cubed (continued)

- Use models to discuss questions like the following.
  1. How many faces on a 1-cube?
  2. How many cubes are in a 3-cube?
  3. How many painted faces are there on a 3-cube?
- Assign students to groups. Encourage them to build models and label painted faces with colored stick-ons as they complete *Practice 19.*
- Check groups' progress when the chart has been completed for the 3-cube. Encourage groups to look for patterns at this point.
- Help groups find the general rule for the N-cube.

Have students check their own work and include *Practice 19* in student portfolios.

Have groups challenge one another with varying numbers for N.

Challenge the students to extend the chart to a 10-cube or a 100-cube.

Compare the resulting theoretical probability against actual drawing and rolling (experimental probability).

Lee Bailey and Tony Tangario, Brevard County

N-Cubed

Directions: *Use models to organize the information for the following situations in the chart below. The first one is done for you.*

1. If you paint a 1-cube and toss it, what is the probability that the top face will be painted?

____

2. Paint a 2-cube, cut it into eight 1-cubes, select one of them at random, and toss it. What is the probability that the top face will be painted?

____

3. Paint a 3-cube, cut it into twenty-seven 1-cubes, select one of them at random, and toss it. What is the probability that the top face will be painted?

____

4. Extend the probability chart to a 6-cube. Use the chart to find the rule for an N-cube.

Cube Size	# of Smaller Cubes	Total Faces	# of Painted Faces	Probability of Top Face Painted
1-cube	1	1 X 6 = 6	6	1
2-cube				
3-cube				
4-cube				
5-cube				
6-cube				
N-cube				

Activity 20

A Penny for Your Thoughts

The student will

- add and multiply whole numbers and decimals to solve real-world problems, using appropriate methods of computing (MA.A.3.3.3).
- describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs (MA.D.1.4.1).

exponent, power

Explain to the class that Julian and Sean are best friends who make a wager to see who can save the most money. Each boy uses a savings plan he thinks best. At the end of each week the contest leader is treated to pizza. Whoever has saved the most money at the end of 30 days is declared the winner.

Pennies or markers. Scientific calculator. Graphing calculator. Poster boards.

- Present students with the following situation.

  Julian and Sean are friends who compete in every way imaginable. If one is good, the other is better. If one has a lot, the other has more. They are both equally serious about their money.

  Each boy has his own method of saving money. Sean pitches pennies, saying they are worthless. Julian saves them, taking time to pick up every penny he sees.

A Penny for Your Thoughts (continued)

Sean boasts of being able to save \$10 per day, simply because he gets an allowance and earns money by doing odd jobs. He challenges Julian to see who has the most money at the end of 30 days. Julian is able to save a penny the first day, two pennies the second day, four the third day, eight the fourth day, and so on. Sean suggests a weekly check-in, with the leader being treated to pizza.

- Give each student at least 100 pennies or markers, with a maximum of 3,000 per class.
- Tape three poster-size calendars, showing days 1-10, 11-20, and 21-30, to a table.
- Ask students to place the pennies on the appropriate place on the calendar to demonstrate Julian's daily savings.
- Set up similar calendars to show Sean's daily savings using pretend \$10 bills.
- With the incomplete calendars in place, ask students to work at their desks, and use calculators to complete the *Table of Values* for the 30 days (see *Practice 20*).
- Compare the savings of each boy on Day 7, Day 14, Day 21, and Day 28 (as shown in the *Table of Values*).
- Work with the class as needed to find the rules (functions) representing the amount of accumulated savings at the end of x days.

The equations are: Sean: $y = 10x$
Julian: $y = .01\ (2^x - 1)$.

Note: Point out to students that on the calculator the power function should be used first as $y = (2^x - 1)\ .01$.

A Penny for Your Thoughts (continued)

- Discuss the following questions.

1. Who pays for the pizza at the end of each week?
2. Who has the most money at the end of 30 days?
3. What difference would be made if Sean were able to save $100 per day? $1000 per day?

Rate groups or individuals on their completion of the *Table of Values* and responses to the questions.

Have students summarize what they learned from this activity.

Using a graphing calculator, graph the functions representing Sean and Julian's accumulated savings.

Suggested range:
$x_{\text{min.}}=1$ $y_{\text{min.}}=1$
$x_{\text{max.}}=30$ $y_{\text{max.}}=500$
$x_{\text{scl.}}=1$ $y_{\text{scl.}}=1$

Use the trace function to examine the point of intersection of the two graphs.

Joyce Hawkins, Broward County

Table of Values

Directions: *Use a calculator and another sheet of paper to complete this table for 30 days. Answer the questions that follow.*

Day	Amount Julian Saves Daily	Julian's Accumulated Savings	Sean's Accumulated Savings
1	$ .01	$ .01	$10
2	$ .02	$ .03	$20
3	$ .04		
4			
5			
6			
7			

1. What are the rules (functions) representing the accumulated savings for Julian and Sean? Let x = ***the number of days*** and y = ***the accumulated savings***.

____

____

2. Who pays for the pizza at the end of each week?

____

Table of Values (continued)

3. Who has the most money at the end of 30 days?____________________

4. What difference would be made if Sean were able to save $100 per day?

____

____

5. What difference would be made if Sean were able to save $1000 per day?

____

____

Activity 21

Save the Patient

The student will

- describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs (MA.D.1.4.1).
- add and multiply whole numbers and decimals to solve real-world problems using appropriate methods of computing (MA.A.3.3.3).

Present the following to the class.

> Sly Clyde undergoes surgery. A nurse gives him 8 mg of medicine at 8:00 a.m. Sly Clyde will be released from the hospital as soon as the quantity of the drug falls below 1 mg. When will Sly Clyde be released if at the end of each hour after the drug is given there remains 75% of the amount present at the beginning of the hour?

Calculators. Graphing calculators (optional). Copies of *Practice 21*.

- Introduce the Attraction and provide each student or groups of students with a copy of *Practice 21*.
- Discuss the problem and address any questions. Ask students to use the information provided to complete *Practice 21*.

Save the Patient (continued)

Observe and rate students on their ability to use various calculator functions.

Evaluate students on how well they completed *Practice 21*.

Ask students to calculate the following.

> If the nurse is asked to administer an additional 1 mg of medication each hour after the initial dose, will the amount of the drug continue to decrease or will it stabilize at a certain point?

Suzanne Johnston

Save the Patient — Practice 21

Hospital Release Table

Directions: *Read the story and follow the directions below.*

Sly Clyde undergoes surgery. A nurse gives him 8 mg of medicine at 8:00 a.m. Sly Clyde will be released from the hospital as soon as the quantity of the drug falls below 1 mg. When will Sly Clyde be released if at the end of each hour after the drug is given there remains 75% of the amount that was present at the beginning of the hour?

Complete the following steps to find a pattern and write an equation that fits the problem. Record your answers in the table at right. Let x = the number of hours.

Elapsed Hours — **Mg of Medicine**

1. After 1 hour 8 X .75 = __________
2. After 2 hours (8 X .75) X .75 = __________
3. After 3 hours (8 X .75) X .75 X .75 = __________
4. After 4 hours __________ = __________

Let h = number of elapsed hours and y = number of mg present.

5. Then y = __________.

Complete the table as far as needed to find out when Sly Clyde can be released.

6. At approximately what time can the patient be released?

____

Number of Elapsed Hours	Mg Medicine Present

Hospital Release Table (continued)

7. List the data from the chart as ordered pairs.

Graph the data on a coordinate plane or use a graphing calculator.

If using a graphing calculator, enter the following:

Set range to:
x min = 0
x max = 9.5
x scl = 1
y min = 0
y min = 8
y scl = 1

Enter the data as a scatter plot, and then verify by drawing the graph. Once this is done, use the trace and zoom functions to pinpoint more accurately when the patient can be released.

Activity 22

Shaq's Magic Shoe Size

BLUEPRINT

The student will

- collect, organize, and display data in a variety of forms, including tables (MA.E.1.3.1).
- interpret data that has been collected, organized, and displayed in charts, tables, and plots (MA.E.1.4.1).
- describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs (MA.D.1.4.1).
- apply properties of two-dimensional figures by using a rectangular coordinate system (MA.C.3.4.2).
- understand and apply the concepts of range and central tendency (mean, median, and mode) (MA.E.1.3.2). (See Extension.)

KEYWORDS

coordinate grid, line of best fit, slope, y–intercept

ATTRACTION

Explain to students that by using basketball star Shaquille O'Neal's size 20 shoe as a model, they will attempt to find the relationship between foot length and forearm length. Students will take their own measurements and use this data in their study.

TOOLS

Tape measures. Graph paper and/or graphing calculator. Poster with outline of Shaquille O'Neal's 41 cm long, size 20 shoe. Copies of *Practice 22A* and *22B*.

STEPS

In this activity students will investigate the linear relationship between a person's foot length and forearm length. Students will collect and organize data, determine line of best fit, investigate slope and y-intercept, and use an equation to make predictions.

Shaq's Magic Shoe Size (continued)

- Have students measure their foot length and their forearm length from elbow to the tip of the index finger.
- Students exchange this data, compile class measurements, and complete *Practice 22A*.
- Discuss the following questions with the class.
  1. Looking at the graph of the data, what characteristics do you see?
  2. Does there appear to be a relationship?
  3. What is the meaning of slope of the line?
  4. What is the meaning of a y–intercept?
  5. Is there a line of "best fit"?
  6. Are there any restrictions on the graph? (negative arm length, etc.)
  7. Can you make any predictions?

After determining a pattern for arm length based on foot length, ask students to predict the arm length of Shaquille O'Neal based on his foot length of 41 cm.

Students work individually to complete *Practice 22B*. They then check their own work with at least two other students, make corrections, and hand in for a recorded grade.

Using a graphing calculator, have students graph the "best fit" equation and determine if the equation fits the data on the coordinate graph.

Shaq's Magic Shoe Size (continued)

EXTENSION

Students may perform another data collection and analysis independently, studying another relationship (e.g., age vs. height). (See *Practice 22B.*)

Find measures of central tendency (range, mean, median, and mode) for data collected regarding foot and arm length.

Linda Ferriera and Joyce Wiley, Pinellas County

Shaq's Magic Shoe Size Practice 22A

Discovering Foot Length

Directions: *Follow the directions and answer the questions below.*

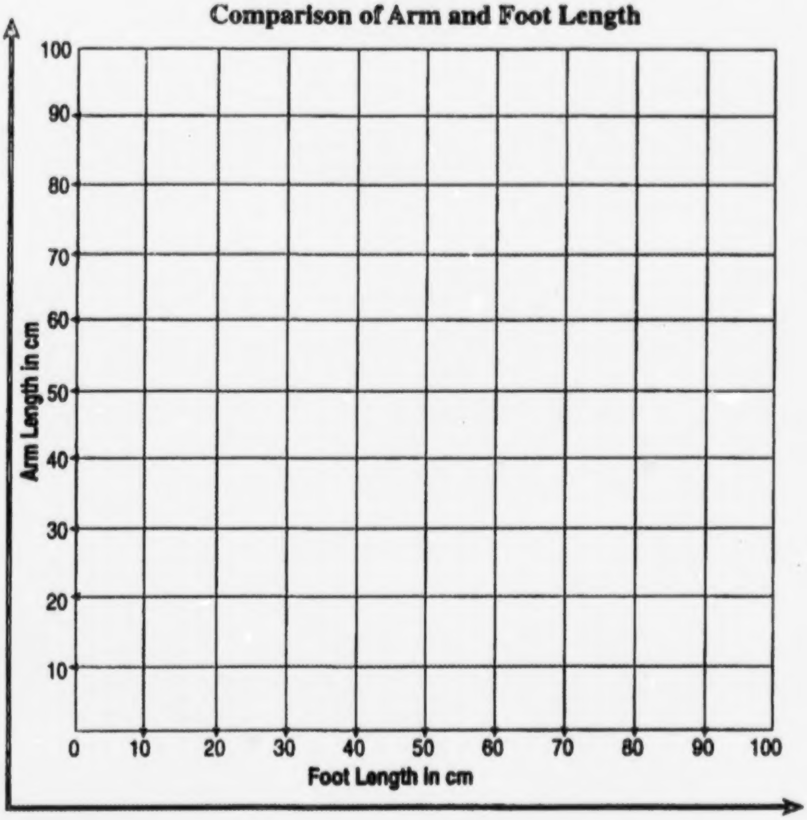
1. Using tape measures, measure foot length and length of arm from elbow to tip of index finger in centimeters.
2. Record data in the table for 20 of your classmates.

Data Record

Trial	Name	Foot Length (X)	Arm Length (Y)
#1			
#2			
#3			
#4			
#5			
#6			
#7			
#8			
#9			
#10			
#11			
#12			
#13			
#14			
#15			
#16			
#17			
#18			
#19			
#20			

Discovering Foot Length (continued)

3. Graph the data on the coordinate grid below.



Comparison of Arm and Foot Length

4. Is there a relationship between foot length and arm length? If so, what is it?

5. Draw a line of best fit.

Discovering Foot Length (continued)

6. Determine the slope of the line. What happens to the arm length as the foot length increases?

____

____

What are the restrictions on the relationship?

____

____

7. Can the foot length be 0? ______________________
8. Can the foot length be a negative number? ______________________
9. What is the minimum value for foot length? ______________________
10. What is the maximum value for foot length? ______________________
11. Write an equation to determine arm length from foot length.

y = ____

12. Using your equation, predict the arm length of Shaquille O'Neal if his foot length is 41 cm.

____

Shaq's Magic Shoe Size

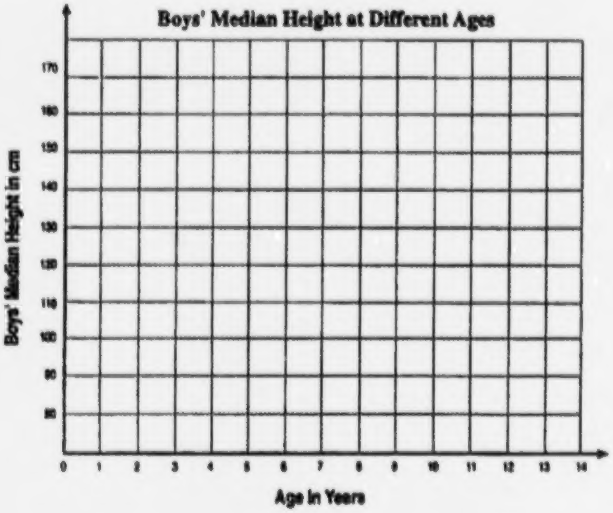
Practice 22B

Investigating Linear Relationships

Directions: *Solve the problems below. Write the correct answers in the tables given below.*

1. Graph the following data.

Median Height of Boys	
Age (x)	**Height (y)**
2	87 cm
3	95 cm
4	103 cm
5	110 cm
6	116 cm
7	122 cm
8	127 cm
9	132 cm
10	138 cm
11	143 cm



Boys' Median Height at Different Ages

2. Draw a best fit line.

3. What is the slope? ______________________________

4. What is the y–intercept? ______________________________

5. What are the restrictions?

____

____

6. Predict the height of a 12-year-old.

____

Investigating Linear Relationships (continued)

7. Predict the height of a 16-year-old.

8. What are the variables that could change the slope for the information after 11 years of age?

9. Why is it impossible to find an exact equation?

10. Summarize the information contained in the table.

Activity 23

Math and Music

The student will

- describe, analyze, and generalize functions using variables and graphs (MA.D.1.4.1).
- interpret data that has been collected, organized, and displayed in charts, tables, and plots (MA.E.1.4.1).
- add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing (MA.A.3.3.3).
- determine the impact on a function when a constant is changed (MA.D.1.4.2).
- understand and apply the concepts of range and tendency (mean, median, and mode) (MA.E.1.3.2).

data, mean

Present the following to the class.

> You received a CD player for your birthday, and you want to start your own CD collection. You must decide between shopping at local stores and joining a record club. Which is the better deal?

Ads from newspapers or magazines for record clubs (e.g., BMG, Columbia House, RCA). Catalogs from a record club showing offerings and prices. Graph paper. Graphing calculator. Copies of *Practice 23A*, *23B*, and *23C*.

- Introduce the project and have each cooperative group make a list of targeted CDs for comparison shopping. Ask students to bring in ads for record clubs. If students belong to a record club, ask them to bring in a catalog.

Math and Music (continued)

- Have students share the different prices from local stores, make a list of CD prices, and calculate the mean price by completing *Practice 23A*.
- Read and discuss various club agreements. List any questions to ask before joining a club.
- Examine record club catalogs. Have each group choose a record club and find the average price of a CD through that club by completing *Practice 23B*.
- Help students build a table that compares costs of various numbers of CDs by completing *Practice 23C*.
- Help students write equations to represent the cost of x CDs for the local store and through a record club.

Example:

Let x = number of CDs
and y = total purchase price

Suppose that the mean price of a CD including tax at a local store is \$17.31. Then the equation is $y = 17.31x$.

Suppose there is an introductory offer at a record club of 10 CDs for \$13.20 and that the mean price of a CD including shipping/handling and tax but not counting the introductory offer is \$20.11.

Then the equation for the cost of x records is

$$y = 20.11\,(x - 10) + 13.20$$
$$y = 20.11x - 201.10 + 13.20$$
$$y = 20.11x - 187.90$$

Note: The minimum value of x in this equation is 10.

Math and Music (continued)

- Students may use graph paper or a graphing calculator to graph the equations. Discuss the labeling of the axes and whether there is an intersection point for the two lines.
- Have students complete the questions in *Practice 23C*. Then have groups present their findings to the class.

Evaluate student performance by checking the group worksheet, group presentation, and individual essay.

Find the best deal by comparing membership charges for credit-card companies, long-distance telephone charges, or memberships to a sports/exercise club. Use a graphing calculator to graph the equations.

Nancy S. Murphy, Brevard County

CD Prices at Local Stores

Directions: *Record the CD names and prices in the chart below. Then find the mean price of a CD at the local store.*

Local Stores

List Prices

Name of Item	Qty.	Unit Price	Amount
		TOTAL	

Mean Price of a CD $ ____________

Tax $ ____________

Total Mean Price $ ____________

1. Write an equation representing the cost y of x CDs purchased at local stores.

____

CD Prices through Record Clubs

Directions: *Record the CD names and prices in the table below, and find the mean price of a CD through the record club.*

Record Club Catalog

List Prices

Name of Item	Qty.	Unit Price	Amount
		TOTAL	

Mean Price of a CD $ ____________

Tax $ ____________

Sub-total $ ____________

Shipping and Handling $ ____________

Total Mean Price $ ____________

1. Write an equation representing the cost of x CDs purchased through the record club. (Caution: Consider special offers).

____

CD Cost Comparison

Directions: *Use the information in the tables from Practice 23A and 23B to complete the table below comparing the accumulated costs of CDs. Then answer the questions that follow.*

Accumulated Costs of CDs

Total # of CDs	At Local Stores	Through Record Club
* 10		
20		
40		
60		
80		
100		
200		
** X		

* Include any special offers in the cost.
** Enter general equations calculated in class.

1. What is the minimum cost and minimum number of CDs you must purchase to meet the requirements of the record club?

____

2. What would that same purchase cost at the local store?

____

CD Cost Comparison (continued)

3. Will the club prices and the store prices ever be the same?

4. Is one deal (store or club) always the best deal?

5. What are the pros and cons of each deal?

6. Would you choose to purchase CDs through a record club or a local store?

Why?

Activity 24

Al Gebra and Daughters Box Company

BLUEPRINT

The student will

- use algebraic problem-solving strategies to solve real-world problems involving linear equations (MA.D.2.3.2).
- describe a wide variety of relationships and functions through models, such as manipulatives, tables, and equations (MA.D.1.3.1).
- represent and apply geometric properties and relationships to solve real-world problems (MA.C.3.3.1).
- understand and describe how the change of a figure in such dimensions as length, width, and height affects its other measurements such as area, surface area, and volume (MA.B.1.3.3).
- add, subtract, and multiply whole numbers, decimals, and mixed numbers to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator (MA.A.3.3.3.).

KEYWORDS

cost analysis, dimensions, surface area, volume, waste

ATTRACTION

Explain to the class that Al Gebra and Daughters Box Company must fill orders for boxes meeting certain specifications. Today they will work in teams representing the four divisions of the company to plan and build the best box to fill an order.

TOOLS

5" x 8" note cards or card stock. Scissors. Tape. Ruler. Calculators. Copies of *Practice 24A*, *24B*, and *24C*.

Al Gebra and Daughters Box Company (continued)

Present students with the following situation.

- Al Gebra, after looking at many successful companies, decided to divide his box company into four balanced teams. Each team would be headed by one of his four daughters. Division I was headed by his oldest daughter, Prob M. Solving. Division II, III, and IV were headed by his other three daughters Commun I. Kations, Reas N. Ing, and Conne X. Tions, respectively. Al Gebra decided to have each team work on the orders simultaneously.
- Each team is responsible for generating the information described on *Practice 24A* and for completing *Practice 24B*. (Explain to the students that the *Practice 24B* is a general worksheet and only the needed information should be completed for each order.)

Note: The Production Cost Analysis may be done for any or all orders. (See Practice 24C.*) The author suggests that all teams work on Order #1 to learn the process and then complete Order #4 for team assessment purposes. Order #2 would be good for individual assessment. Order #3 would make a good extra-credit problem or a non-assessed final group problem.*

Use Order #4 for team assessment.

Use Order #2 for individual assessment because it can be solved in a variety of ways, including trial and error, the quadratic formula, or with a graphing calculator. The cost analysis can be included or excluded from assessments.

Al Gebra and Daughters Box Company (continued)

EXTENSION

The *Production Cost Analysis, Practice 24C,* may be used as an extension for any or all of the orders.

Use community resources. Invite a speaker from a factory to discuss production costs, or take a field trip to the facility.

Bob Davis, Charlotte County

Order Information

Directions: *Respond to the statements below for each order.*

1. the quantity of waste per box and the dimensions of the waste pieces
2. the dimensions of the box to meet the order specifications
3. all relevant equations needed by the computer-controlled box-making machine to produce the specified box (e.g., volume, surface area, waste, ratios comparing any two of these)
4. a summary record of the problem-solving process used by the team to satisfy the order specifications
5. a persuasive argument that the box designed by the team met the order specifications
6. a prototype box

Order #1

Build an open box from a piece of 5" by 8" cardboard by cutting squares of equal size from each corner. The box is to have the maximum volume possible with dimensions given to the nearest ¼".

Order #2

Build an open box from a piece of 5" by 8" cardboard by cutting squares of equal size from each corner. The box is to have a surface area as close as possible to the combined area of the four squares cut from the corners. All dimensions are to be given to the nearest ¼".

Order #3

Build an open box from a piece of 5" by 8" cardboard by cutting squares of equal size from each corner. The box is to have the smallest ratio possible between the surface area and volume (i.e., build the box that has the smallest quantity of cardboard per cubic inch of volume). (Note: Disregard any waste consideration.) All dimensions are to be given to the nearest ¼".

Order #4

Build an open box from a piece of 5" by 8" cardboard by cutting squares of equal size from each corner. The box is to have the number of square inches of waste as close as possible to the number of cubic inches of volume in the box. All dimensions are to be given to the nearest ¼".

Order Analysis

Directions: *Complete only the columns necessary for the particular order.*

AL GEBRA AND DAUGHTERS
BOX COMPANY

Order Analysis Worksheet

Order Number:________

Box Options	Length	Width	Height	Volume	Surface Area	Amount of Waste
A						
B						
C						
D						
E						

Equations: Complete only those equations needed for the order.

Volume = ____________________

Surface Area = ____________________

Waste = ____________________

Ratio of ________ to ________ = ____________________

Order Description: ____________________________________

____

____

____

The dimensions of the best box for this order are ____________________

Al Gebra and Daughters Practice 24C

Production Cost Analysis

Directions: *Construct the best box for the order, and do a production cost analysis. Determine a unit selling price for the box. Use these guidelines: Overhead cost = 20% of the production cost. The profit = 10% of the total cost.*

AL GEBRA AND DAUGHTERS
BOX COMPANY

Production Cost Analysis Worksheet

Order Number: ________

Box Option	Material Cost 1¢ per sq. in. of surface area	Waste Cost .5¢ per sq. in.	Cutting & Assembly .75¢ per linear in. of cutting	Total Production Cost	Subcontractor Price $1.25 per sq. in. of surface area	Selling Price
A						
B						
C						
D						
E						

Material Cost = ____________

Total Production Cost = ____________

Waste Cost = ____________

Subcontractor Price = ____________

Cutting and Assembly = ____________

1. What is the selling price of the box including production cost, overhead, and profit? ____________________________

2. Who should produce the boxes, Al Gebra or the subcontractor?

   ____

   Why? ____________________________

Appendices

Answer Keys

Practice 1B (p. 4)

1. 34
2. $12.95
3. $440.30
4. 5
5. $1.49
6. $7.45
7. 13
8. $.79
9. $10.27
10. 16
11. $.89
12. $14.24
13. 5
14. $2.29
15. $11.45
16. 34
17. $.98
18. $33.32
19. $76.73
20. $517.03

Practice 1C (p. 5)

1. $440.30
2. 4
3. 6
4. $33.32
5. $43.41
6. 13 hrs. 30 min.
7. $3.97
8. Answers will vary.

Practice 2B (p. 10)

Answers will vary based on student's grades.

Practice 3 (pp. 14-15)

Correct answers will be determined by the teacher.

Practice 5 (p. 21)

Answers will vary.

Practice 6A (p. 25)

Answers will vary.

Practice 6B (p. 26)

Answers will vary.

Practice 12 (p. 58)

4. 25 (4m + 13)
   100m + 325
5. 100m + 125
6. 100m + 125 + D
7. 2 (100m + 125 + D)
   200m + 250 + 2D
8. 200m + 2D + 210
9. 50 (200m + 2D + 210)
   10,000m + 100D + 10,500
10. 10,000m + 100D + 10,500 + y
11. 10,000m + 100D + y

Practice 15 (p. 70)

1. F
2. D
3. B
4. A
5. G
6. E
7. C
8. no
9. Answers will vary.

Practice 16 (p. 74)

Fraction Card Denominator	Number of Fractions	Fraction Town Population
2	1	1
3	2	3
4	3	6
5	4	10
6	5	15
7	6	21
8	7	28
9	8	36
10	9	45
11	10	55
12	11	66
13	12	78
14	13	91
15	14	105

Answer Keys (continued)

Practice 18 (p. 82)

Accumulated 12-Week Salary		
Week	Lee	[illegible]
1	$ 0	$ 320
2	$ 0	$ 490
3	$ 240	$ 660
4	$ 480	$ 830
5	$ 720	$ 1,000
6	$ 960	$ 1,170
7	$ 1,200	$ 1,340
8	$ 1,440	$ 1,510
9	$ 1,680	$ 1,680
10	$ 1,920	$ 1,850
11	$ 2,160	$ 2,020
12	$ 2,400	$ 2,190

1. Lee's Lunchorama
2. yes; during week 9

Practice 19 (p. 85)

1. $6/6 = 1$
2. ½
3. ⅓
4.

Cube Size	# of Smaller Cubes	Total Faces	# of Painted Faces	Probability of Top Face Painted
1-cube	1	1 X 6 = 6	6	1
2-cube	8	48	24	½
3-cube	27	162	54	⅓
4-cube	64	384	96	¼
5-cube	125	750	150	⅕
6-cube	216	1,296	216	⅙
n-cube	n^3	n^3 X 6	n^2 X 6	$1/n$

Practice 20 (pp. 90-91)

Day	Amount Julian Saves Daily	Julian's Accumulated Savings	Sean's Accumulated Savings
1	$.01	$.01	$10
2	$.02	$.03	$20
3	$.04	$.07	$30
4	$.08	$.15	$40
5	$.16	$.31	$50
6	$.32	$.63	$60
7	$.64	$1.27	$70

1. Sean: $y = 10x$
   Julian: $y = .01\,(2x - 1)$
2. Julian
3. Julian
4. Sean would save $3,000. Julian would still be the winner.
5. Sean would save $30,000. Julian would still be the winner.

Practice 21 (pp. 95-96)

1. 6
2. 4.5
3. 3.375
4. $(8 \times .75) \times .75 \times .75 \times .75 = 2.531$
5. $8\,(.75)^x$
6. 4:00 p.m.

Practice 22A (pp. 100-102)

Answers will vary.

Practice 22B (pp. 103-104)

Answers will vary based on placement of line.

Practice 23A (p. 108)

Answers will vary.

Practice 23B (p. 109)

Answers will vary.

Practice 23C (pp. 110-111)

Answers will vary.

Practice 24A (p. 116)

Correct answers will be determined by the teacher.

Practice 24B (p. 117)

Answers will vary.

Practice 24C (p. 118)

Answers will vary.

Sunshine State Standards Correlation Matrix

Math Activities	Page No.	Standards and Benchmarks																											
		MA.A.								MA.B.			MA.C.					MA.D.					MA.E.						
		1.3.3	1.4.1	1.4.2	1.4.4	2.4.1	3.3.2	3.3.3	3.4.3	1.2.2	1.3.3	1.4.3	1.3.1	2.4.1	3.3.1	3.3.2	3.4.2	1.3.1	1.4.1	1.4.2	2.3.1	2.3.2	1.3.1	1.3.2	1.4.1	2.3.1	2.3.2	2.4.1	3.2.1
1. Magic Valley	1							●																					
2. Earn Your Way through School	7							●													●				●				
3. How Many Bears?	11											●			●														●
4. The Shadow Knows	17											●																	
5. Nutty Putty	19				●					●		●																	
6. The Big Drip	23											●											●						
7. Step Forward and Take a Bow	27		●	●				●	●																				
8. High Rollers	31				●																								
9. Algebra War	35						●		●												●								
10. Language Concentration	47								●												●								
11. Magic Squares	51							●											●										
12. A Magical Mathematical Birthday	55																		●			●							
13. The Human Coordinate Plane	59															●			●										
14. Picture This	63													●		●			●										
15. Where in the Venn?	67			●														●											
16. This Town Ain't Big Enough for All of Us...or Is It?	71	●	●	●		●													●										
17. What's the Blur?	75																		●										
18. Summer Earnings/Pick a Job	79							●											●						●				
19. Probability Cubed	83												●						●							●	●	●	
20. A Penny for Your Thoughts	87							●											●										
21. Save the Patient	93							●											●										
22. Shaq's Magic Shoe Size	97																●		●				●	●	●				
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Appendix D

Multimedia Bibliography

Software

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Alge Blaster 3. Eugene, OR: Learning Services.

Alge Blaster Data Discs. Eugene, OR: Learning Services.

Alge Blaster Plus. Eugene, OR: Learning Services.

Algebra Concepts. Eugene, OR: Learning Services.

Algebra 1: First Semester. Gainesville, FL: Computer Centerline.

Algebra Plus Volume One. Gainesville, FL: Computer Centerline.

Algebra Plus Volume Two. Gainesville, FL: Computer Centerline.

Algebra Proposer. Gainesville, FL: Computer Centerline.

Algebra Series. Eugene, OR: Learning Services.

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The Cruncher. Eugene, OR: Learning Services.

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Escape from Algebra. Gainesville, FL: Computer Centerline.

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High School Math Competency. Mansfield, OH: Opportunities for Learning.

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Integers/Equations. Waltham, MA: Cambridge Development Laboratory, Inc.

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Knowledge Quest: Mathematics. Elgin, IL: Educational Resources.

Let's Celebrate Math. Mansfield, OH: Opportunities for Learning.

Major League Math. Eugene, OR: Learning Services.

Math Blaster Mystery: The Great Brain Robbery. Eugene, OR: Learning Services.

Math Bundle. Elgin, IL: Educational Resources.

MathCheck. Elgin, IL: Educational Resources.

Math Diagnostic Inventory. Waltham, MA: Cambridge Development Laboratory, Inc.

Math Sequences. Waltham, MA: Cambridge Development Laboratory, Inc.

Math Sleuths. Seattle, WA: Videodiscovery.

Math Trek 7, 8, & 9. Elgin, IL: Educational Resources.

Math Trek, 10, 11, & 12. Elgin, IL: Educational Resources.

Mathematics VII. Eugene, OR: Learning Services.

Mathematics VIII. Eugene, OR: Learning Services.

NumberMaze Challenge. Eugene, OR: Learning Services.

Operation Neptune. Eugene, OR: Learning Services.

Order of Operations. Waltham, MA: Cambridge Development Laboratory, Inc.

Probability Lab. Gainesville, FL: Computer Centerline.

Probability Toolkit. Eugene, OR: Learning Services.

The Quarter Mile Math Programs. Eugene, OR: Learning Services.

SAT Preparation. Gainesville, FL: Computer Centerline.

Understanding Algebra. Elgin, IL: Educational Resources.

Understanding Equations. Elgin, IL: Educational Resources.

Understanding Exponents. Elgin, IL: Educational Resources.

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Videos

What are Variables? Mansfield, OH: Opportunities for Learning.

Videodiscs

Modumath: Algebra. Chicago, IL: Society for Visual Education.

Printed Materials

Alternative Assessment. Mansfield, OH: Opportunities for Learning.

Practical Math. Mansfield, OH: Opportunities for Learning.

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Problem of the Week Series. Mansfield, OH: Opportunities for Learning.

Wizard Math Books. Mansfield, OH: Opportunities for Learning.

Wizard Math: Word Problems. Mansfield, OH: Opportunities for Learning.

Sources

Cambridge Development, Inc.
86 West Street
Waltham, MA 02154

Computer Centerline of Florida
3955 NW 29th Lane
Gainesville, FL 32606-6652

Dataflex Corporation
861 Hercules Avenue N.
Clearwater, FL 34625

Appendix D

Educational Resources
1550 Executive Drive
Elgin, IL 60121-1900

Learning Services
P.O. Box 10636
Eugene, OR 97440-2636

Opportunities for Learning
941 Hickory Lane
Mansfield, OH 44901-8103

Society for Visual Education
6677 N Northwest Highway
Chicago, IL 60631

Sunburst Communications
101 Castleton Street
Pleasantville, NY 10570

Videodiscovery, Inc.
1700 Westlake Avenue, Suite 911
Seattle, WA 98109

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